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MONTE CARLO SIMULATION OF AN AAA SYSTEM USING THE OBSERVER MODE--ETC(U)
AUG 79 M M VIKMANIS, R S KOU F33615-79-C-0500
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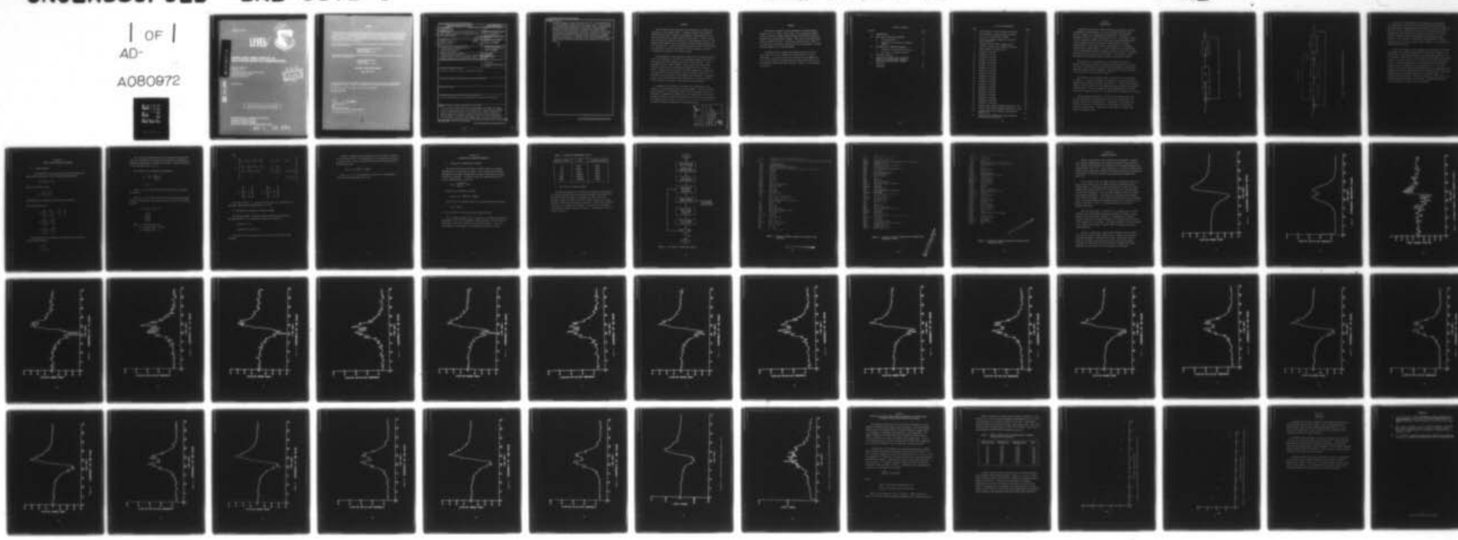

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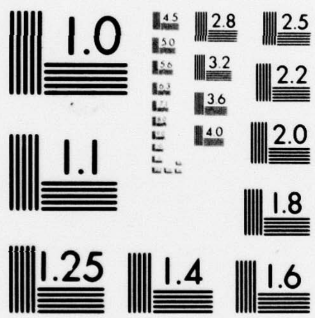

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AUGUST 1979

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FOR THE COMMANDER

CHARLES BATES, JR.
Chief
Human Engineering Division
Air Force Aerospace Medical Research Laboratory

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This technical report documents the development of a Monte Carlo simulation of the observer model. The original model outputs were the ensemble mean and standard deviation of the tracking error. Because some existing weapon system attrition models have resorted to Monte Carlo methods it became necessary to develop a compatible simulation of the observer model. The development of the simulation equations and a detailed description of the statistical properties of the remnant element are given in the report

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along with computer program flow chart and code. It is necessary that the sample ensemble of many single Monte Carlo time histories converge to the ensemble mean prediction of the observer model. Program outputs depicting the sample ensemble for a wide range of sample sizes are included which show the excellent convergence of the Monte Carlo simulation to the ensemble mean prediction. Simulation results verify that the Monte Carlo simulation of the observer model can be used with confidence in AAA System Attrition Analysis. Monte Carlo simulation outputs are also used as a reference in the curve-fitting parameter identification programs to determine a good trade-off between the cost of large sample sizes and the high statistical variability of low sample sizes.

SUMMARY

This technical report documents the development of a Monte Carlo simulation of the observer model. Luenberger reduced-order observer theory has been used to develop a mathematical representation of the anti-aircraft-artillery (AAA) manned tracking system. Previous reports show the excellent predictive property of this modeling structure. The original model outputs were the ensemble mean and standard deviation of the tracking error. Because some existing weapon system attrition models have resorted to Monte Carlo methods it became necessary to develop a compatible simulation of the observer model.

The development of the simulation equations and a detailed description of the statistical properties of the remnant element are given in the report along with computer program flow chart and code. It is necessary that the sample ensemble of many single Monte Carlo time histories converge to the ensemble mean prediction of the observer model. Program outputs depicting the sample ensemble for a wide range of sample sizes are included which show the excellent convergence of the Monte Carlo simulation to the ensemble mean prediction. Simulation results verify that the Monte Carlo simulation of the observer model can be used with confidence in AAA System Attrition Analysis.

An important application of the Monte Carlo simulation is the analysis of empirical data sample size requirements. Now that observer model parameters are identified directly from the empirical data it is necessary to determine the sensitivity of the algorithms. Monte Carlo simulation outputs are used as a reference in the curve-fitting programs to determine a good trade-off between the cost of large sample sizes and the high statistical variability of low sample sizes.

PREFACE

This report documents a study performed by Systems Research Laboratories, Inc. (SRL), Dayton, Ohio, for the Aerospace Medical Research Laboratory (AMRL), Manned-Systems Effectiveness Division, Manned Threat Quantification Program. This work was performed under Contract F33615-76-C-5001. The Contract Monitor was Mr. Robert E. Van Patten and the Technical Manager was Dr. Dan Repperger. The SRL Project Manager was Mr. Charles McKeag.

The authors wish to extend their appreciation to Ms. Betty Glass (SRL) for helpful comments and ideas during the project development. The authors also wish to thank Mr. Walt Summers and Capt. George Valentino of Manned-Systems Effectiveness Division of the Aerospace Medical Research Laboratory, WPAFB, for many valuable discussions.

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Section I
INTRODUCTION

Luenberger reduced-order observer theory has been used to develop a mathematical representation of the anti-aircraft-artillery (AAA) gunner's response in a compensatory tracking task. This model is described in detail in other references [1], [2]. The observer concept of state estimation combined with linear state variable feedback and a remnant element provides a simple and computationally efficient representation of the human's control characteristics. A parameter identification program based on the least squares curve-fitting method and the Gauss Newton gradient algorithm eliminates the need for parameter tuning and other trial and error procedures. For reference, a brief description of the AAA gun system and model configuration follows.

The basic task of the AAA gunner is to minimize the error in azimuth and elevation between the weapon system gunsight and the target. The error is displayed on an optical sight and alignment is induced by a handcrank control with rate dynamics. Fundamental variables are shown in the closed loop block diagram of Figure 1.

The elements of the observer model are shown in the block diagram of Figure 2. An observer provides an estimate of the states of the AAA System from the visually observed tracking error. A linear state variable feedback control law based on the estimated states represents the gunner's control output. Sources of randomness such as modeling error, observation error, neuromotor noise, etc. are lumped into one element called remnant.

Computer simulation results of the closed loop AAA tracking task with the previously described gunner model are the ensemble mean and standard deviation of tracking error. The purpose of this paper is to describe the details of the Monte Carlo simulation of an AAA System using the observer model. The output of this model is a single time history of tracking error.

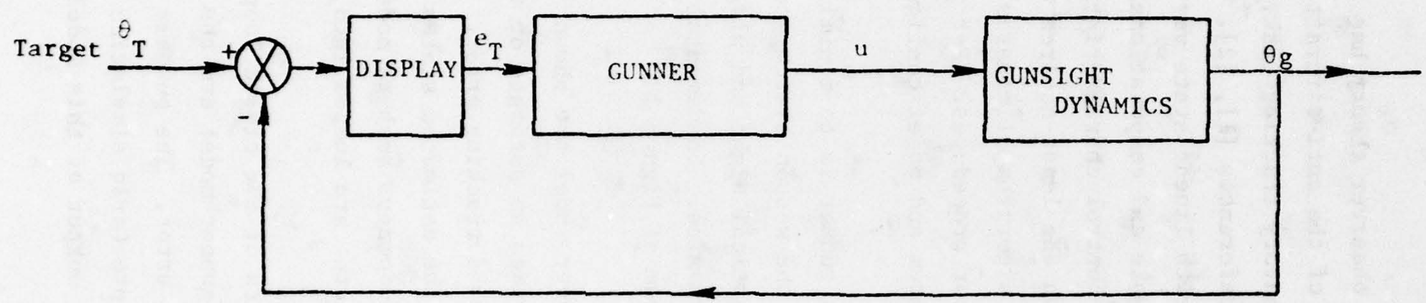


Figure 1. Block Diagram of an AAA Closed Loop System

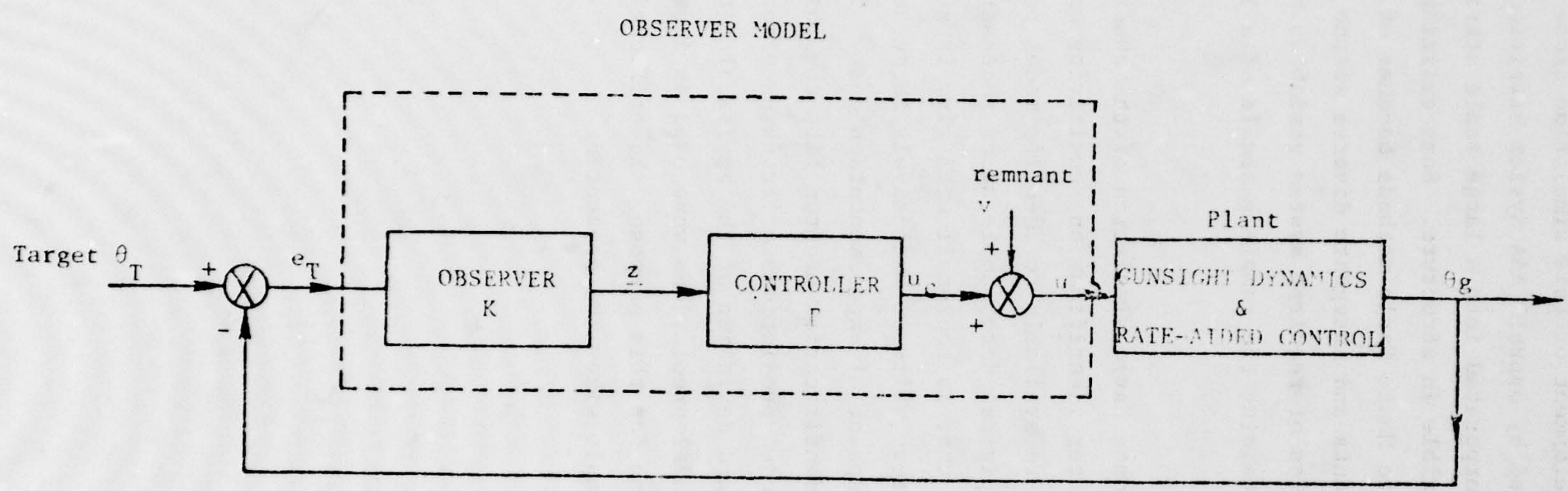


Figure 2. Block Diagram of Observer Model Structure

Motivation for development of this simulation arose primarily from requirements imposed by overall AAA System attrition modeling. The gunner model is incorporated into a large scale attrition model and therefore must be compatible in structure. Some existing attrition programs have resorted to Monte Carlo methods because of the need to simulate nonlinear elements and integrate diverse weapon system functions. The statistics of required system variables can thus be determined easily by obtaining the sample ensemble of a large number of single Monte Carlo runs.

Results from the Monte Carlo simulation of the observer model have been applied in a parameter identification sensitivity analysis. Now that computer programs are available to identify model parameters, it is essential to know empirical data sample sizes necessary for consistent results. A sample size that is too small will give large variability in the estimated parameters while an excessively large sample size is expensive from the standpoint of experimentation cost. The least squares curve-fitting identification program fits the ensemble mean and standard deviation of model predictions of tracking errors to the sample ensemble mean and standard deviation of the empirical data. The ensembled Monte Carlo model predictions were used as input data to the identification algorithms for this purpose. Identification results for a wide number of sample sizes are presented.

Section II

MONTE CARLO SIMULATION EQUATIONS

A. System Equations

The dynamics of the anti-aircraft-artillery gunsight and target motion are given by the following system equations:

$$\dot{\underline{x}} = A\underline{x} + Bu + F\ddot{\theta}_T$$

where $\underline{x}$ is the state vector

$$\underline{x} = \begin{bmatrix} \theta_T - \theta_g \\ \dot{\theta}_T \end{bmatrix}$$

representing the tracking error and the target velocity.

The system matrices are:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The tracking error is displayed to the gunner and is given by the measurement equation:

$$y = C\underline{x}$$

$$C = [1 \quad 0]$$

Now, the system equations are used to design the gunner model. Since the tracking error is available through direct observation, a Luenberger reduced-order observer can be used to obtain an estimate of the remaining state, i.e., $\hat{x}_2$.

The control law is subsequently expressed as

$$u_C = -\begin{bmatrix}\gamma_1 & \gamma_2\end{bmatrix}\begin{bmatrix}y \\ \hat{x}_2\end{bmatrix}$$

$$u = u_C + v$$

Where v is a zero mean Gaussian white noise which represents the remnant.

Finally, it can be shown that the closed loop system (gunsight dynamics, target motion, and gunner model) is given by the following equations

$$\dot{\underline{x}} = A_1\underline{x} + F_1\ddot{\theta}_T + D_1 v$$

$$\underline{x} = \begin{bmatrix}x_1 \\ x_2 \\ x_3\end{bmatrix}$$

Where x_1 is the tracking error y,
x_2 is defined as $\dot{\theta}_T - ky$, and
x_3 is defined as $\dot{\theta}_T - \hat{x}_2$.

And,

$$A_1 = \begin{bmatrix} a_{11} + a_{12}k - b_1(\gamma_1 + k\gamma_2) & a_{12} - b_1\gamma_2 & b_1\gamma_2 \\ (a_{22} - ka_{12})k + a_{21} - ka_{11} & a_{22} - ka_{12} & \\ -(b_2 - kb_1)(\gamma_1 + k\gamma_2) & -(b_2 - kb_1)\gamma_2 & (b_2 - kb_1)\gamma_2 \\ 0 & 0 & a_{22} - ka_{12} \end{bmatrix}$$

$$F_1 = \begin{bmatrix} f_1 \\ f_2 - kf_1 \\ f_2 - kf_1 \end{bmatrix} \qquad D_1 = \begin{bmatrix} b_1 \\ b_2 - kb_1 \\ b_2 - kb_1 \end{bmatrix}$$

The control gains γ_1, γ_2 and the observer gain k are obtained by the parameter identification program [Refs. (1), (2)].

B. Mathematical Description of Remnant Element

The remnant element v represents human psychophysical limitations and modeling error. Its statistical properties are given by,

$$E[v(t)] = 0 \quad \forall t$$

$$E[v(t)v(\tau)] = q(t)\delta(t - \tau)$$

Where q(t) is the covariance function and $\delta(t)$ is the Dirac delta function.

Analysis of empirical data indicates that the standard deviation of the tracking error is due predominantly to the gunner's uncertainty in target motion. Thus the covariance function can be generalized in the following form:

$$q(t) = \alpha_1 + \alpha_2 \hat{\dot{\theta}}_T^2(t) + \alpha_3 \hat{\ddot{\theta}}_T^2(t)$$

Where α_1, α_2, α_3 are nonnegative constants to be determined by the curve fitting algorithm [Ref. (2)].

Section III

DESCRIPTION OF COMPUTER SIMULATION

A. Generation of Random Noise Sequence

The closed loop system equations are discretized for convenient implementation on the digital computer. Thus a random noise sequence representing the remnant element must be generated possessing the given statistical properties. In discrete time, the standard deviation of the zero mean sequence is given by,

$$V(k) = \left[\frac{q(t_k)}{\text{Del}}\right]^{1/2} \eta(k)$$

where $q(t_k)$ is defined as follows:

$$q(t_k) = \alpha_1 + \alpha_2 \hat{\dot{\theta}}_T^2(t_k) + \alpha_3 \hat{\theta}_T^2(t_k)$$

Also, Del is the sampling period of the discretization process and,

$$\eta(k) = N(0,1)$$

is a unit variance, zero mean, Gaussian random variable.

In the computer program, $\eta(k)$ is generated by averaging 12 uniformly distributed (-1/2, +1/2) independent random variables. The statistical properties of $\eta(k)$ were checked and are given by Table 1. For a 100 point sample the convergence is approximately within five percent.

TABLE 1. STATISTICAL PROPERTIES OF $\eta(k)$

Number of Points	Mean	Standard Deviation
16	.36354	.93692
100	.04054	.95703
500	.025437	.96013
1000	.0082841	.96165
2000	-.0027901	.96326
10000	-.00028009	.98910

B. Flow Chart of Computer Program

The computer program for the Monte Carlo simulation was based on the original reduced-order observer model program that generates the mean and standard deviation of tracking error [Refs. (1), (2)]. Necessary changes were made to solve the actual closed loop system equations rather than the expected value form. Code is also included to generate the random noise sequence. A flow chart of the Monte Carlo Computer Program is given in Figure 3. The Fortran IV code is listed in Figure 4.

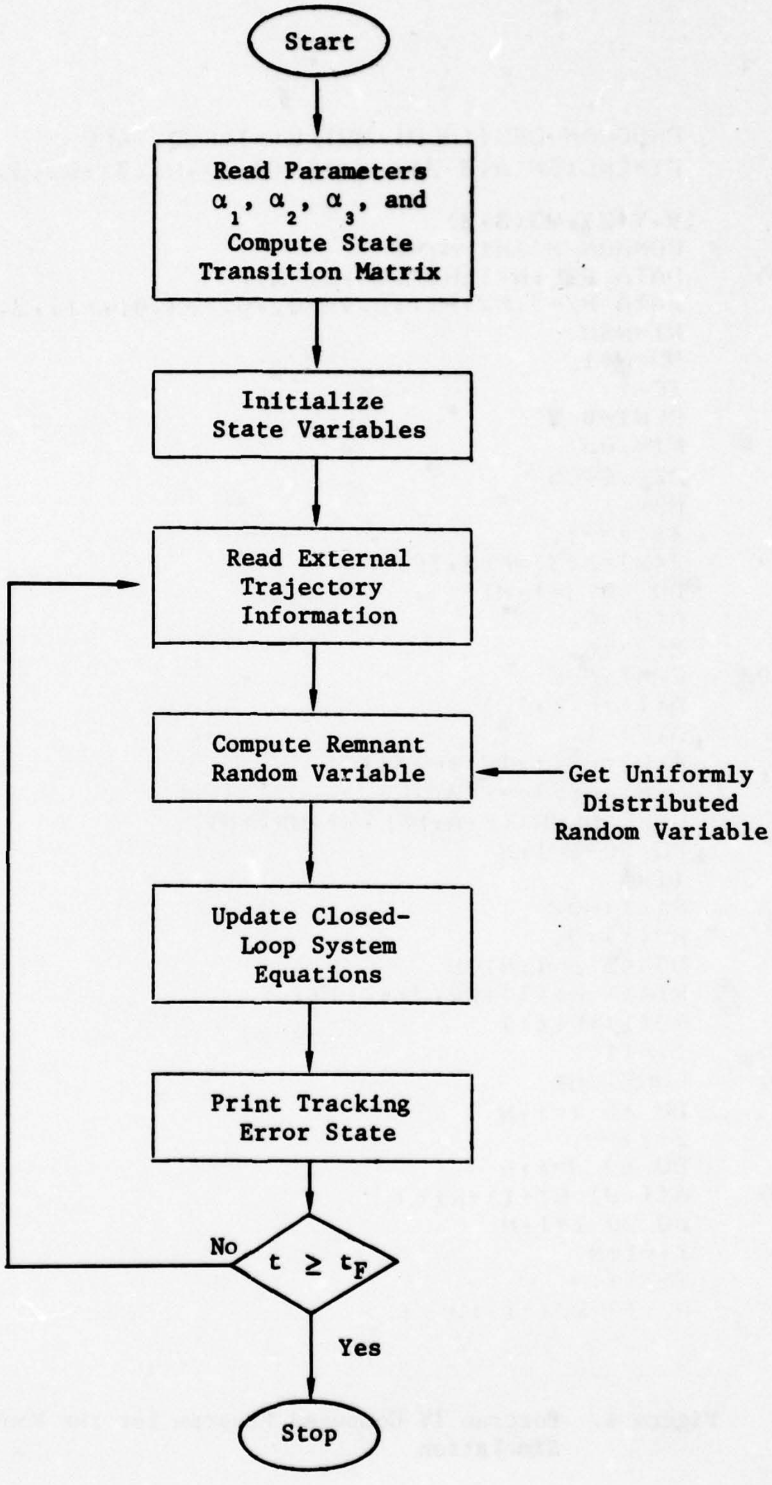


Figure 3. Flow Chart of Simulation Program

```
      PROGRAM OBS(INPUT,OUTPUT,TAPE2,TAPE8)
      DIMENSION A(3,3),Z(3),X(3,3),R1(3),R2(3),W1(3,3),W2(3,3),P(
5,2)
     1 ,Y(3),W3(3,3)
      COMMON/MAIN1/N,N2
      DATA DEL,N,TEND/.04,3,45./
      DATA P/-2.87,-1.,2.94,0.,0.,-3.01,-1.,3.02,0.,0./
      N1=N*N
      N2=N+1
      IC=1
      REWIND 2
      P1=.05
      P2=.0025
      P3=.1
      Z(1)=-1.
      Z(2)=Z(3)=P(3,IC)
      DO 10 J=1,N1
      A(J)=0.
      X(J)=0.
10    CONTINUE
      A(1)=P(1,IC)
      A(7)=1.
      A(2)=-P(1,IC)*P(3,IC)
      A(8)=A(9)=-P(3,IC)
      CALL DSCRT(N,A,DEL,W1,W2,10)
      DO 50 I=1,N
      II=1
      R1(I)=0.
      R2(I)=0.
      DO 45 J=I,N1,N
      R1(I)=R1(I)+W2(J)*Z(II)
      R2(I)=R1(I)
45    II=II+1
50    CONTINUE
      DO 60 I=1,N
      Z(I)=0.
      DO 60 J=1,N
60    A(I,J)=R1(I)*R1(J)
      DO 20 I=1,N
      I1=I+N
      I2=I1+N
20    R1(I)=W2(I1)+W2([illegible]
```

Figure 4. Fortran IV Computer Program for the Monte Carlo Simulation

```
      T=0.
      X(1)=.1*X(5)=X(9)=.01
      Z(1)=Z(2)=Z(3)=0.0
      Y(1)=Y(2)=Y(3)=0.0
1     READ(2,3)C1,P(5,1),P(4,1),C3,P(5,2),P(4,2)
3     FORMAT(6G12.4)
      P4=Z(2)-Z(3)+P(3,IC)*Z(1)
      P5=(P4-PP4)/DEL
      V=(P1+P2*P4*P4+P3*P5*P5)/DEL
      PP4=P4
      CALL RANDOM(GAUSS)
      VK=GAUSS*SQRT(V)
      DO 25 I=1,N
      II=1
      W2(I)=0.
      DO 15 J=I,N1,N
      W2(I)=W2(I)+W1(J)*Z(II)
15    II=II+1
25    CONTINUE
      DO 35 I=1,N
      Z(I)=W2(I)+R1(I)*P(4,IC)
35    CONTINUE
      CALL MULT(W1,X,N,N1,W2)
      DO 40 I=1,N1
40    X(I)=A(I)*V+W2(I)
      SD=SQRT(X(1,1))
      DO 95 I=1,N
      II=1
      W3(I)=0.0
      DO 90 J=I,N1,N
      W3(I)=W3(I)+W1(J)*Y(II)
90    II=II+1
95    CONTINUE
      DO 100 I=1,N
      Y(I)=W3(I)+R1(I)*P(4,IC)+R2(I)*VK
100   CONTINUE
      T=T+DEL
      LK=(T+.001)/DEL
      IF(MOD(LK,25).NE.0)GO TO 48
      PRINT75,T,Y(1)
75    FORMAT(5X,2G12.4)
      WRITE(8,24)T,Y(1)
```

Figure 4. Fortran IV Computer Program for the Monte Carlo Simulation (cont.)

```
24      [illegible]
98      [illegible] TO 500
        GO TO 1
500     CONTINUE
        END
        SUBROUTINE MULT(E,F,L,L1,H)
        DIMENSION E(L1),F(L1),G(9),H(L1)
        DO 10 I=1,L
        II=1
        DO 10 K=1,L
        TEMP=0.
        DO 5 J=I,L1,L
        TEMP=TEMP+E(J)*F(II)
5       II=II+1
        KK=(K-1)*L+I
10      G(KK)=TEMP
        DO 20 I=1,L
        DO 20 K=I,L
        TEMP=0.
        II=K
        DO 15 J=I,L1,L
        TEMP=TEMP+G(J)*E(II)
15      II=II+L
        KK=(K-1)*L+I
20      H(KK)=TEMP
        L2=L-1
        DO 30 I=1,L2
        L3=I+1
        DO 30 J=L3,L
        K1=(I-1)*L+J
        K2=(J-1)*L+I
30      H(K1)=H(K2)
        END
        SUBROUTINE RANDOM(GAUSS)
        C=0.
        DO 55 I=1,12
        C=C+RANF(4.0)
55      CONTINUE
        GAUSS=C-6.0
        END
```

Figure 4. Fortran IV Computer Program for the Monte Carlo Simulation (cont.)

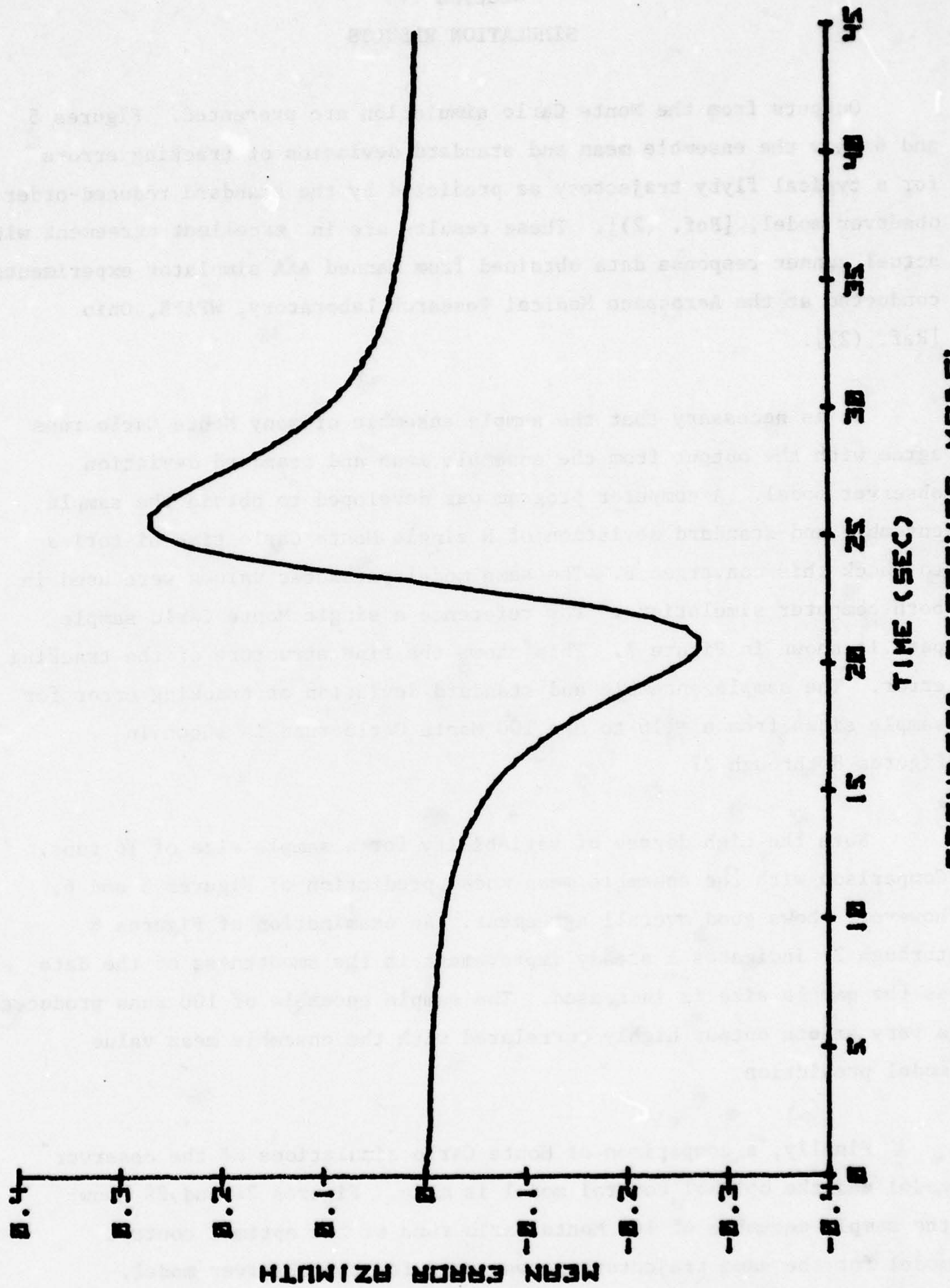
Section IV
SIMULATION RESULTS

Outputs from the Monte Carlo simulation are presented. Figures 5 and 6 show the ensemble mean and standard deviation of tracking errors for a typical flyby trajectory as predicted by the standard reduced-order observer model, [Ref. (2)]. These results are in excellent agreement with actual gunner response data obtained from manned AAA simulator experiments conducted at the Aerospace Medical Research Laboratory, WPAFB, Ohio [Ref. (2)].

It is necessary that the sample ensemble of many Monte Carlo runs agree with the output from the ensemble mean and standard deviation observer model. A computer program was developed to obtain the sample ensemble and standard deviation of N single Monte Carlo time histories to check this convergence. The same model parameter values were used in both computer simulations. For reference a single Monte Carlo sample path is shown in Figure 7. This shows the fine structure of the tracking error. The sample ensemble and standard deviation of tracking error for sample sizes from $n = 16$ to $n = 100$ Monte Carlo runs is shown in Figures 8 through 27.

Note the high degree of variability for a sample size of 16 runs. Comparison with the ensemble mean model prediction of Figures 5 and 6, however, shows good overall agreement. An examination of Figures 8 through 27 indicates a steady improvement in the smoothness of the data as the sample size is increased. The sample ensemble of 100 runs produces a very smooth output highly correlated with the ensemble mean value model prediction.

Finally, a comparison of Monte Carlo simulations of the observer model and the optimal control model is made. Figures 28 and 29 show the sample ensemble of 100 Monte Carlo runs of the optimal control model for the same trajectory as was used for the observer model. Although some differences in model outputs are evident, the overall appearance of the results from the two models is comparable.



STANDARD OBSERVER MODEL
MEAN ERROR AZIMUTH
TIME (SEC)
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
5
10
15
20
25
30
35
40
45

Figure 5.

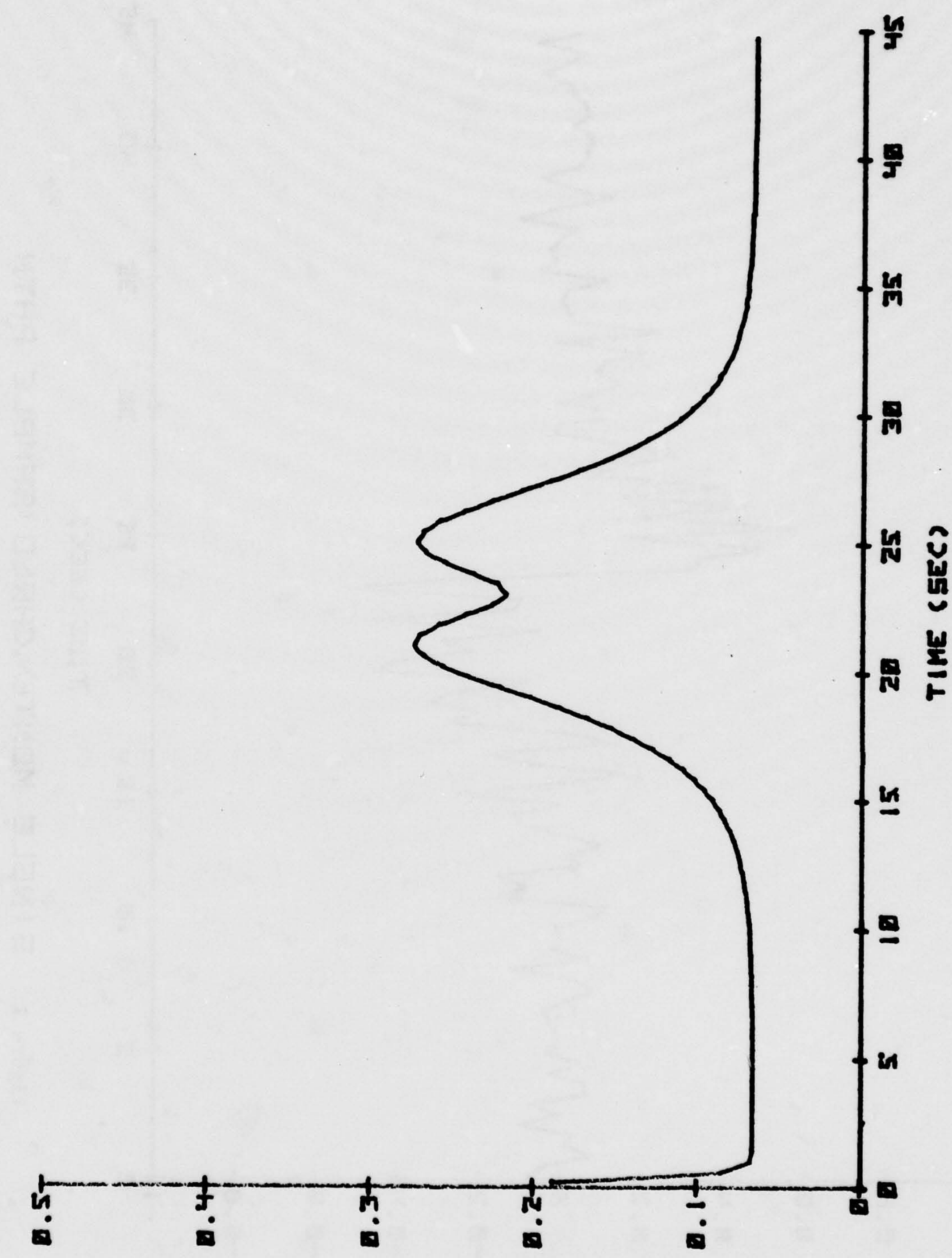


Figure 6. STANDARD OBSERVER MODEL

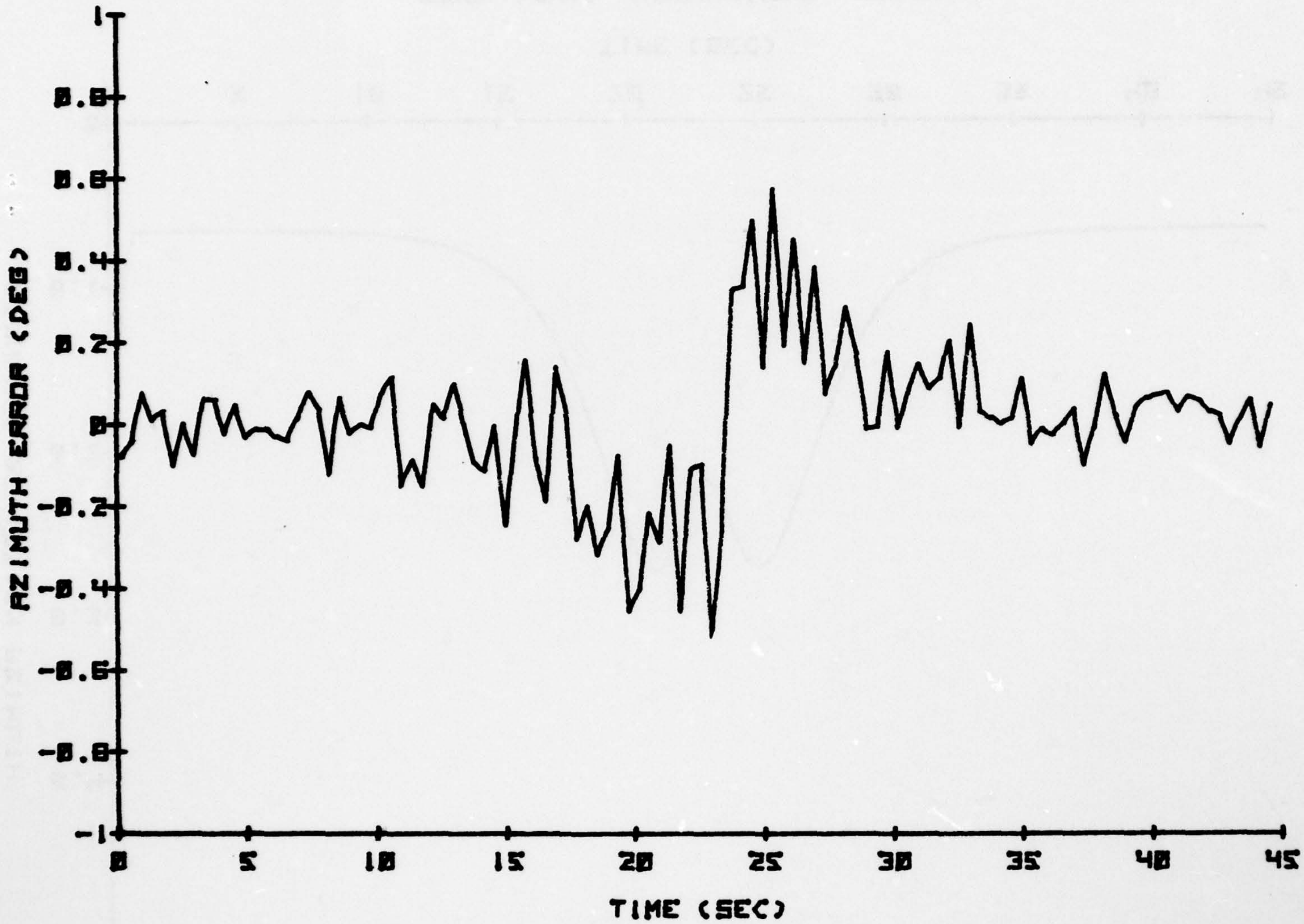


Figure 7. SINGLE MONTE CARLO SAMPLE PATH

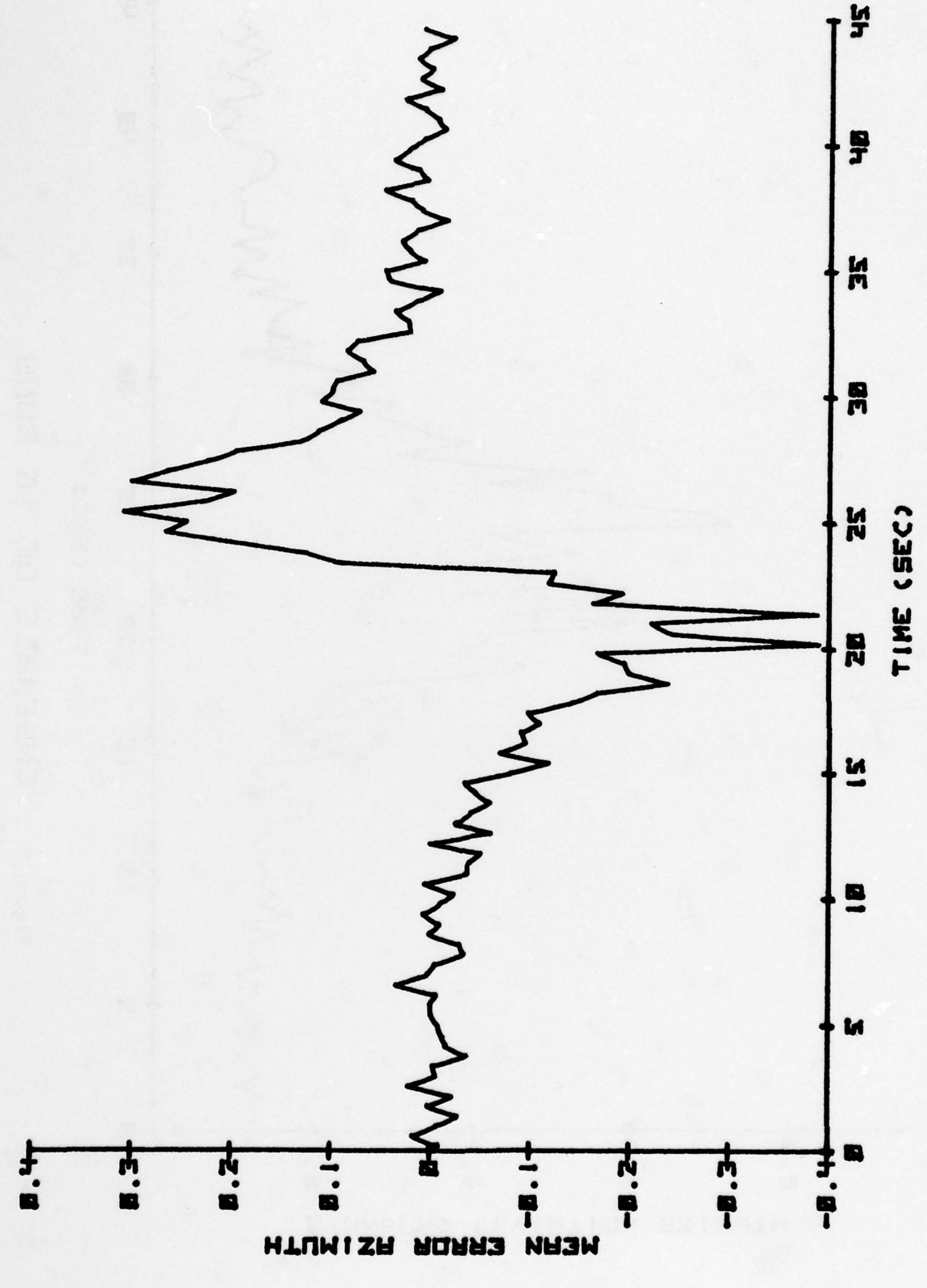


Figure 8. ENSEMBLE OF 16 RUNS

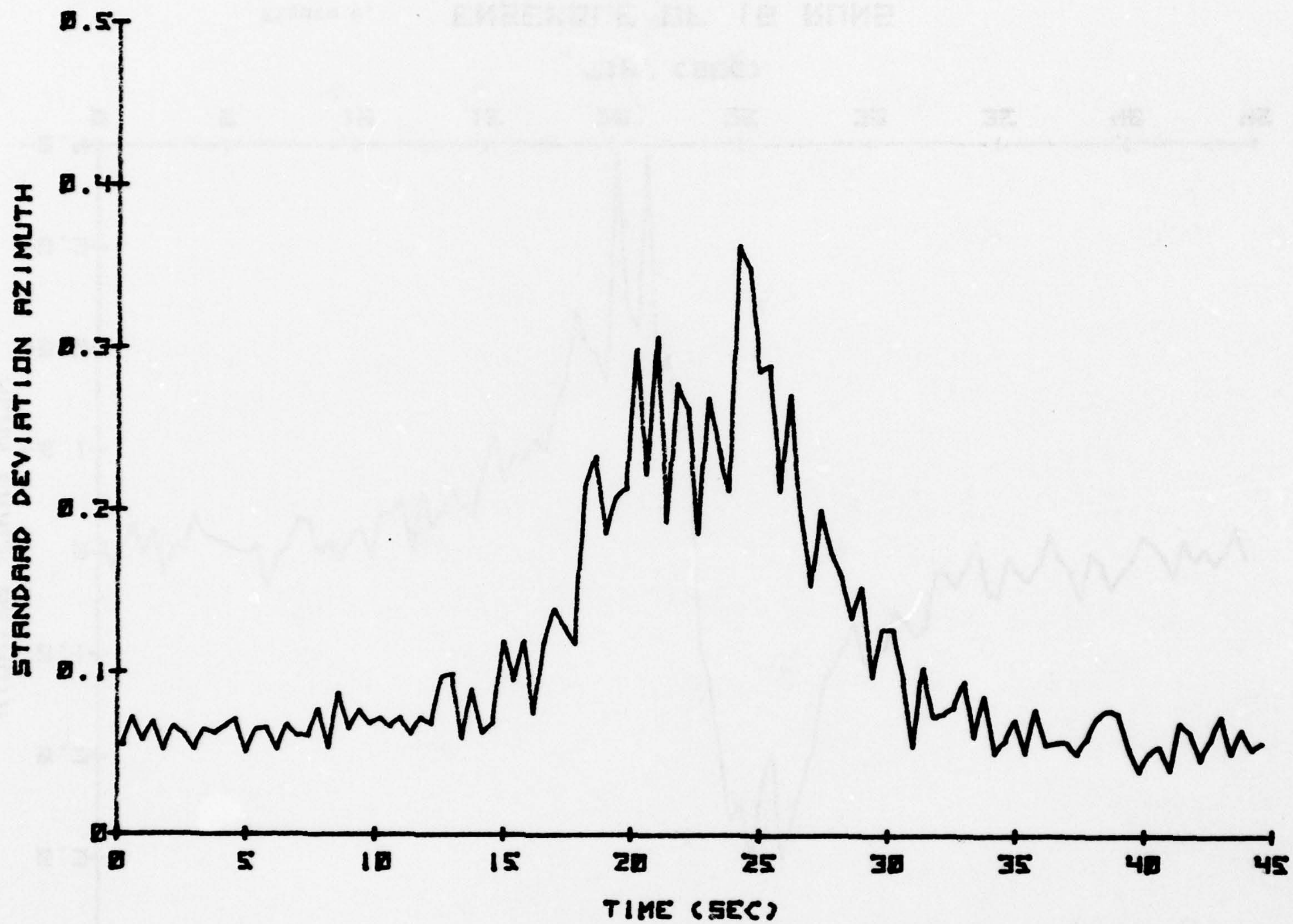


Figure 9. ENSEMBLE OF 16 RUNS

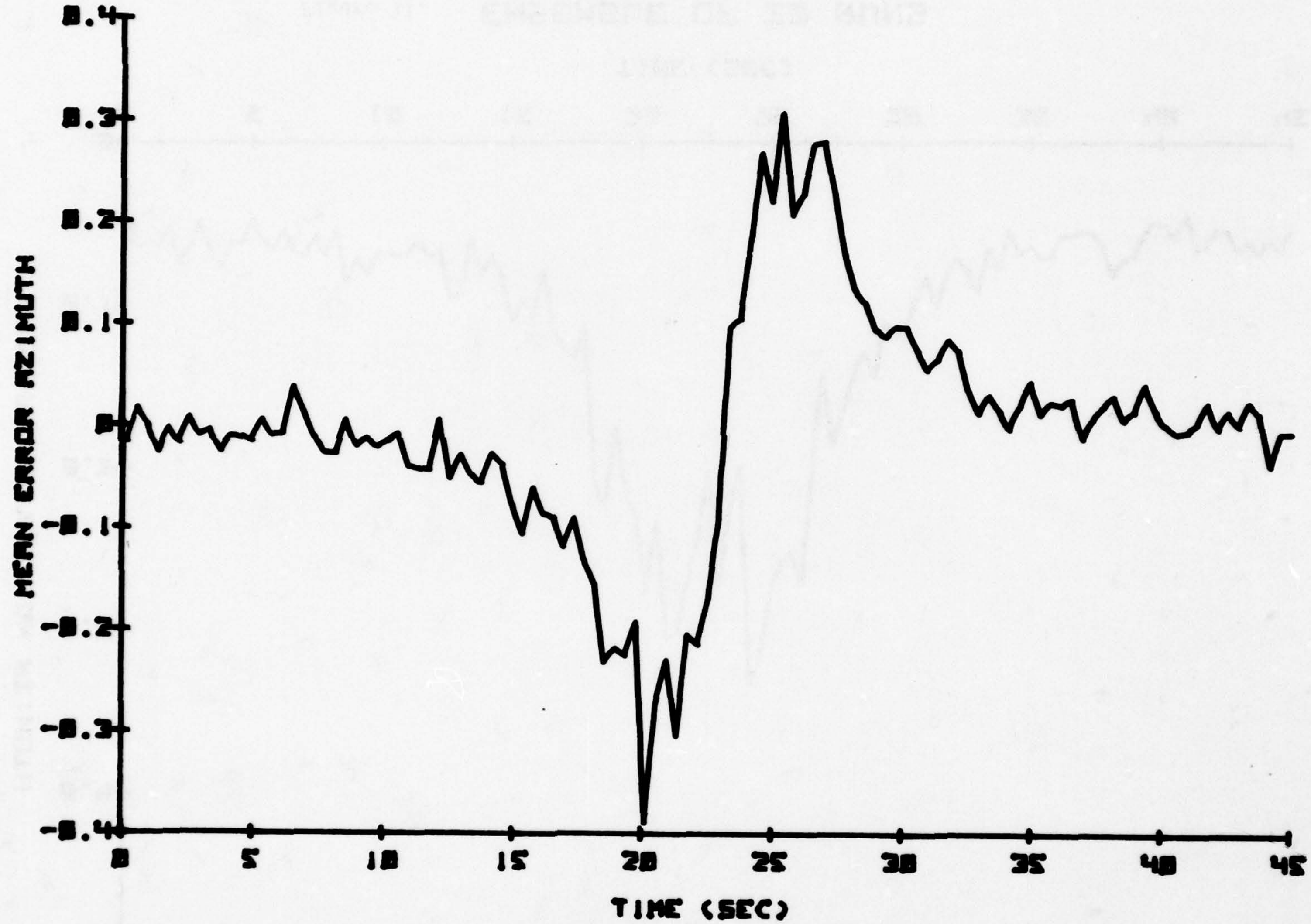


Figure 10. ENSEMBLE OF 20 RUNS

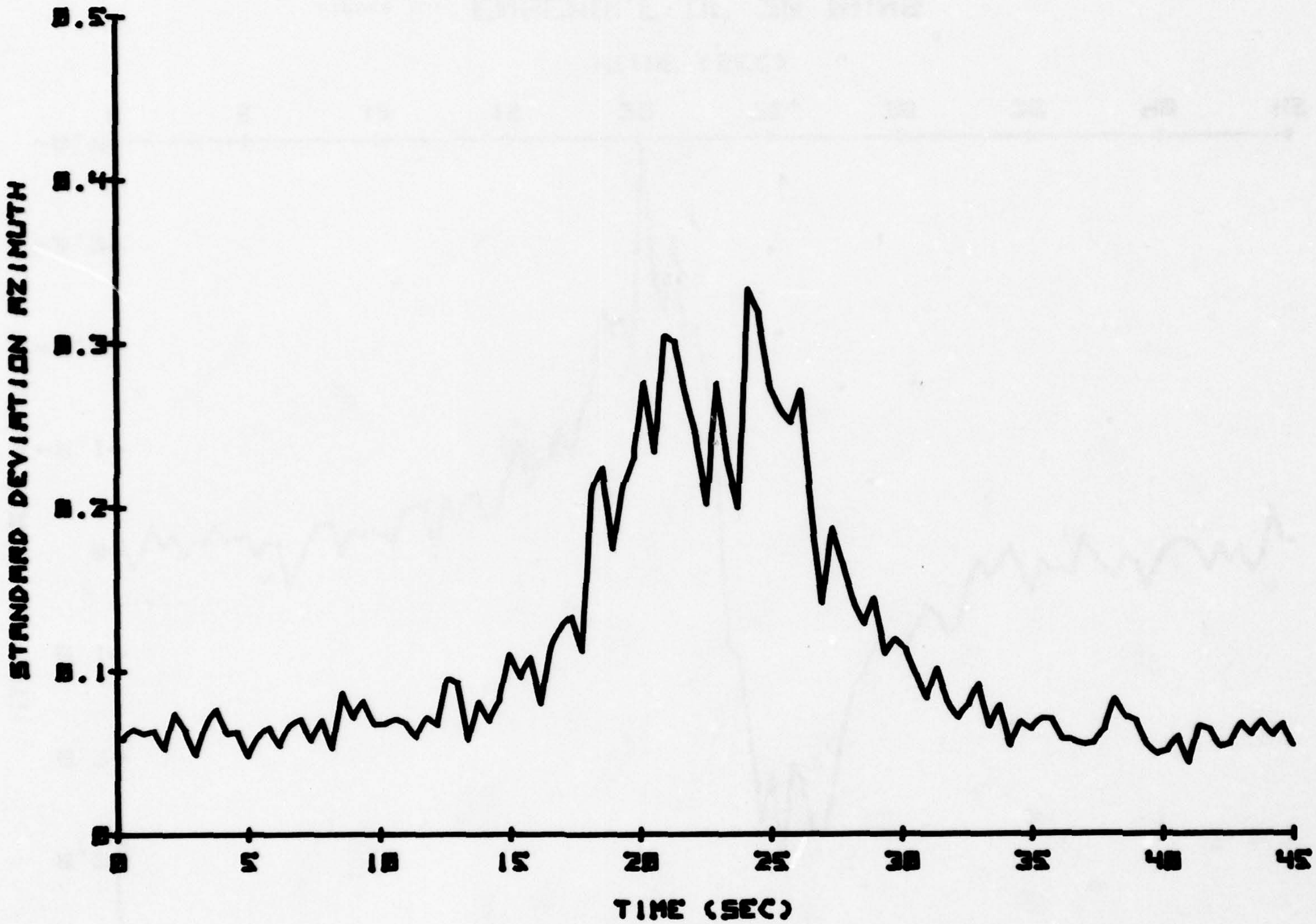


Figure 11. ENSEMBLE OF 20 RUNS

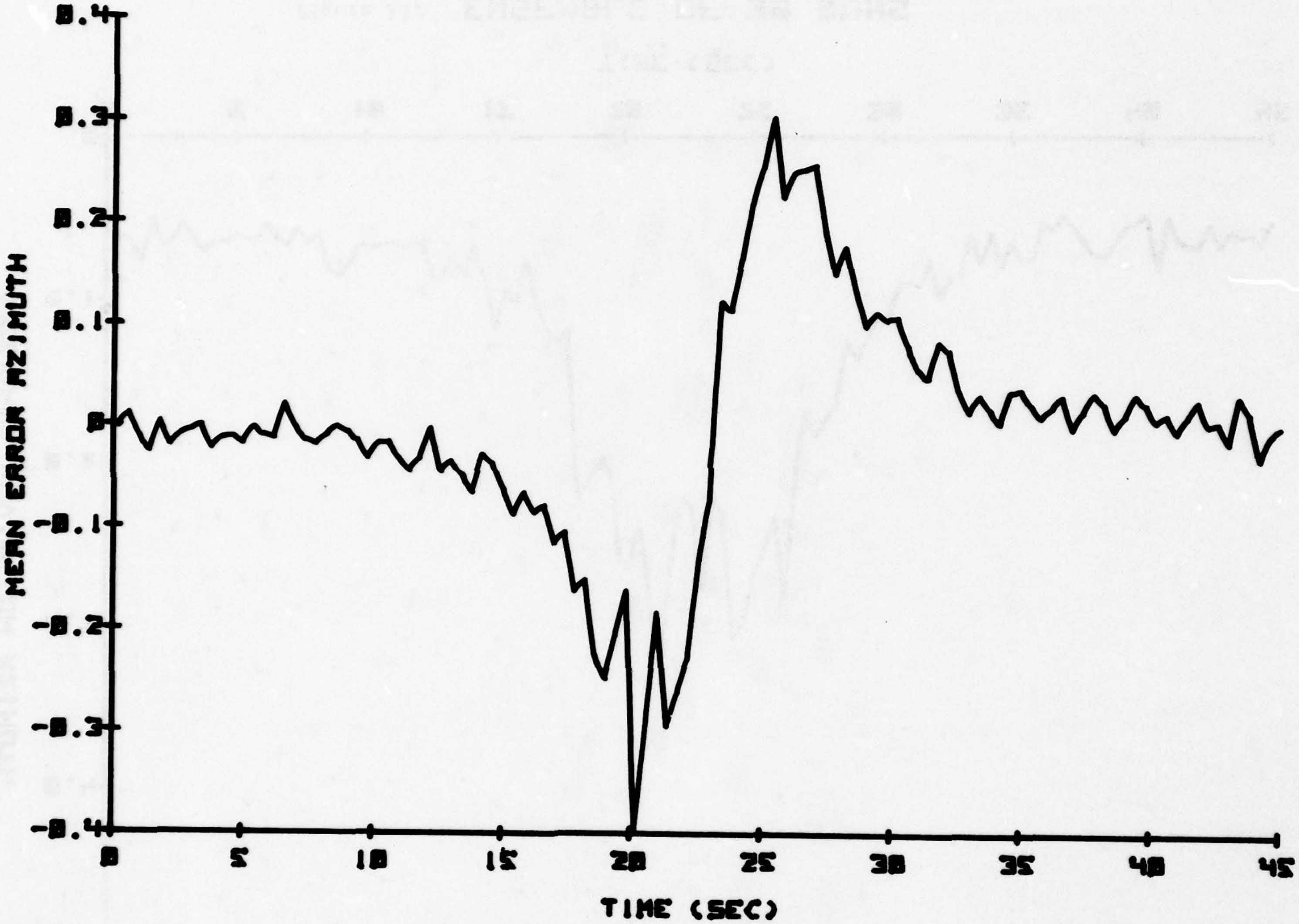


Figure 12. ENSEMBLE OF 30 RUNS

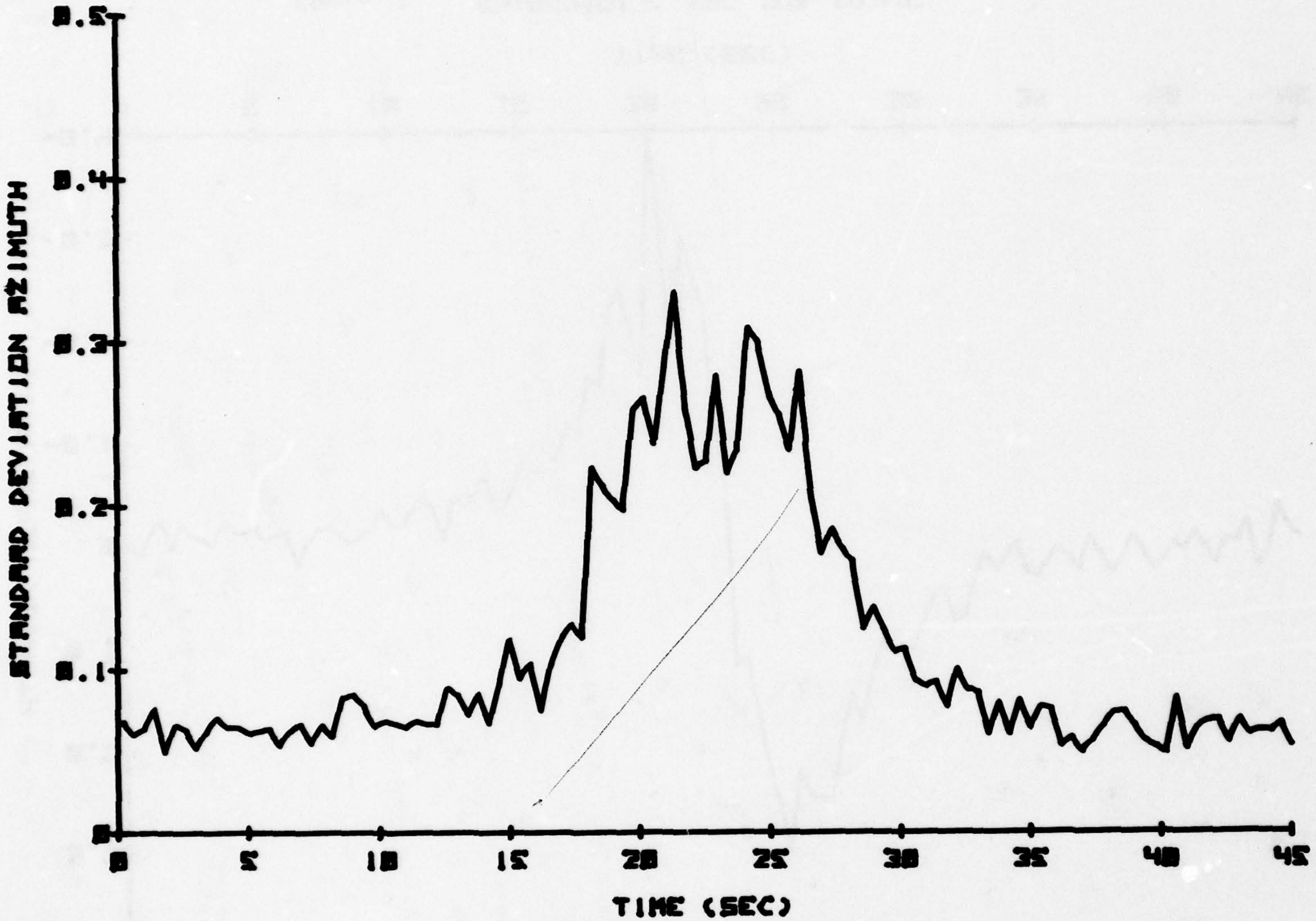


Figure 13. ENSEMBLE OF 30 RUNS

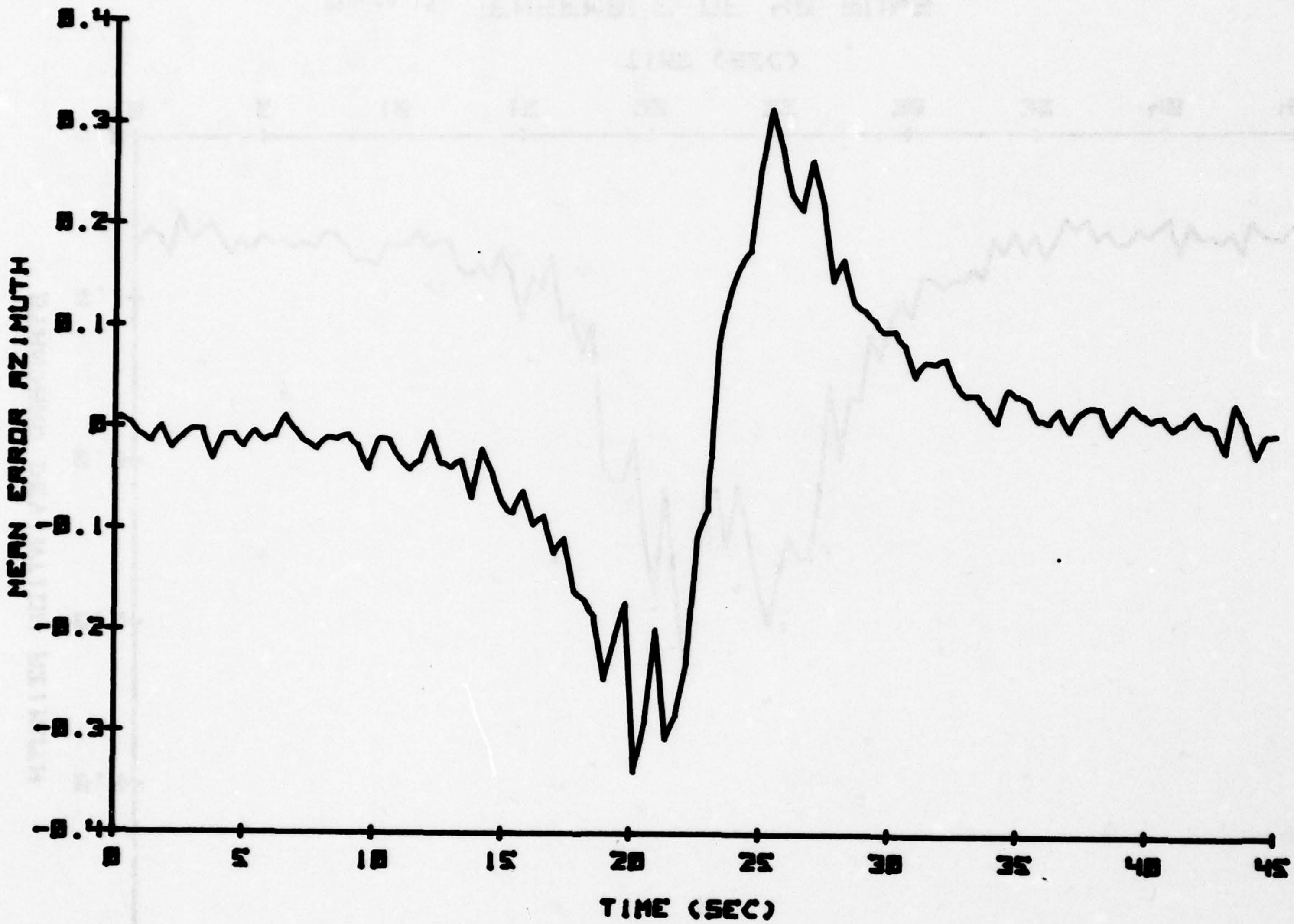


Figure 14. ENSEMBLE OF 40 RUNS

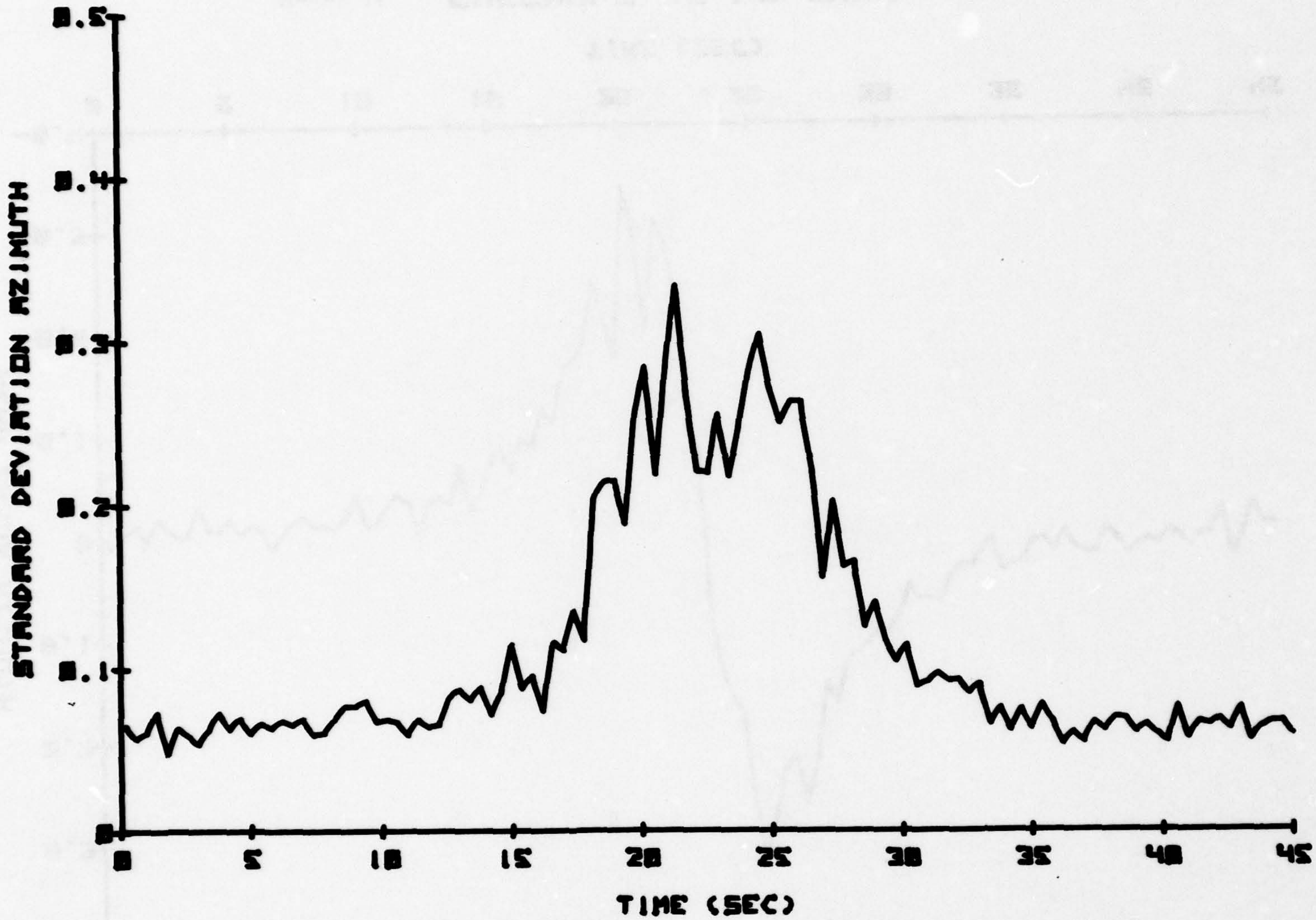


Figure 15. ENSEMBLE OF 40 RUNS

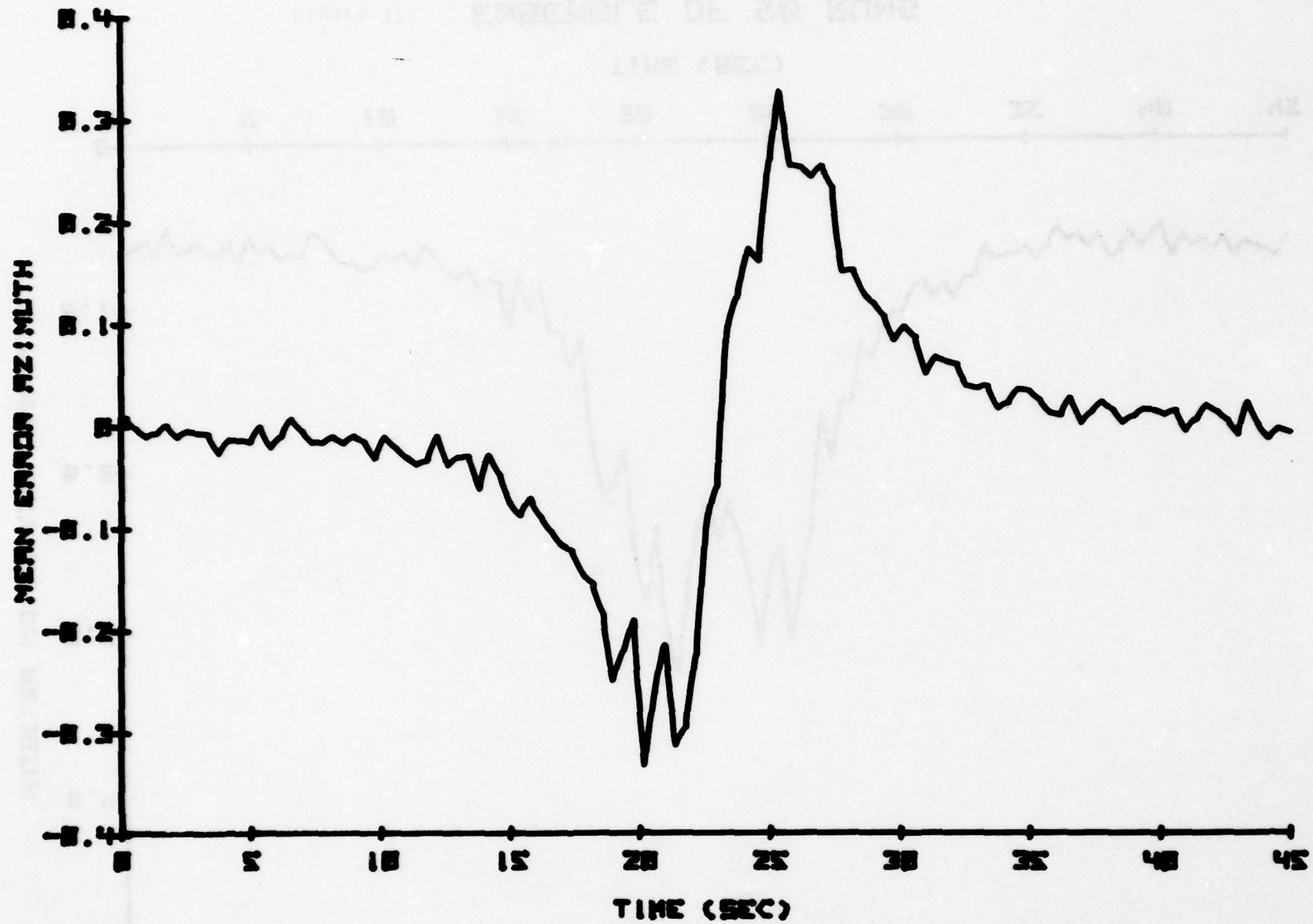


Figure 16. ENSEMBLE OF 50 RUNS

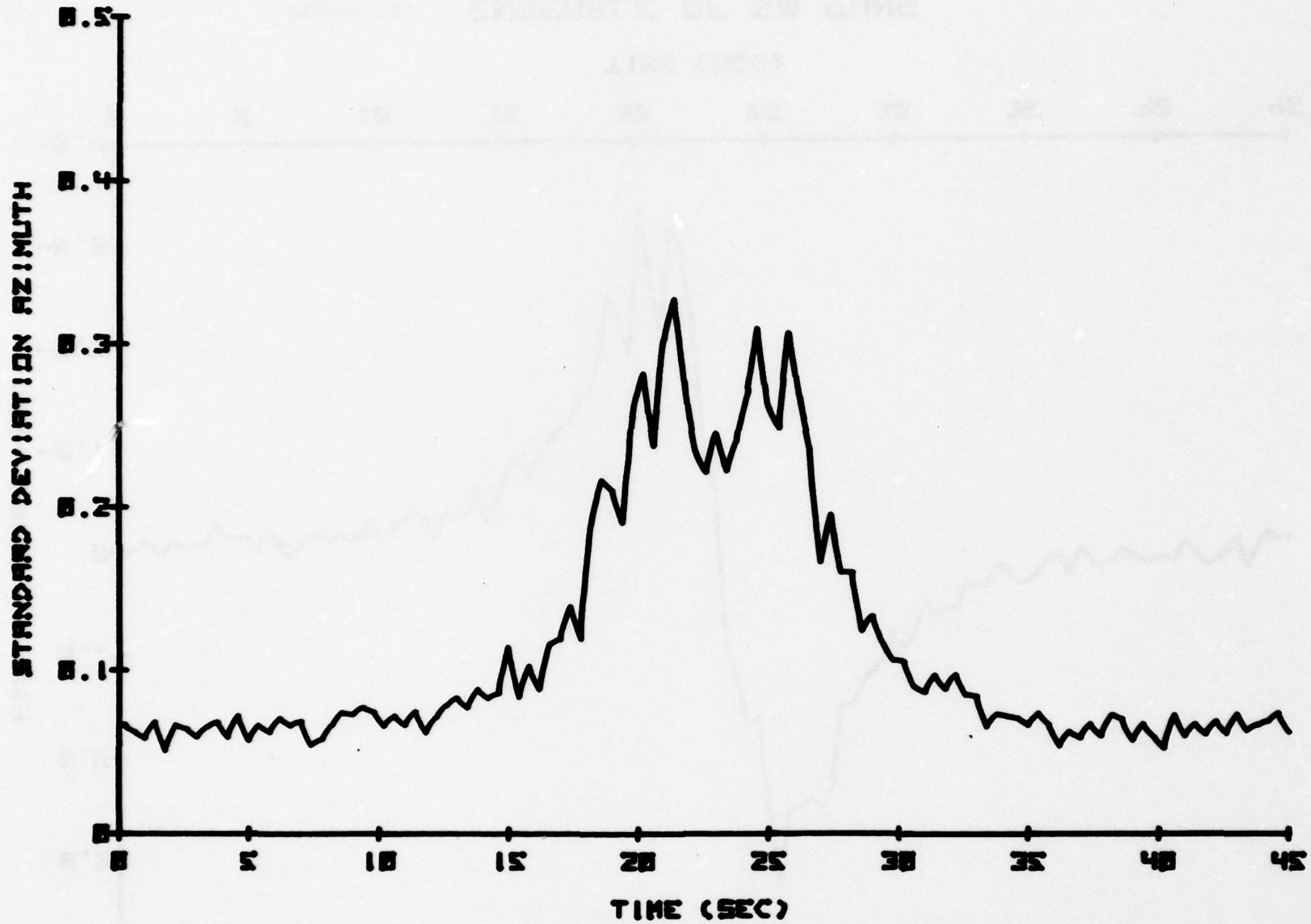


Figure 17. **ENSEMBLE OF 50 RUNS**

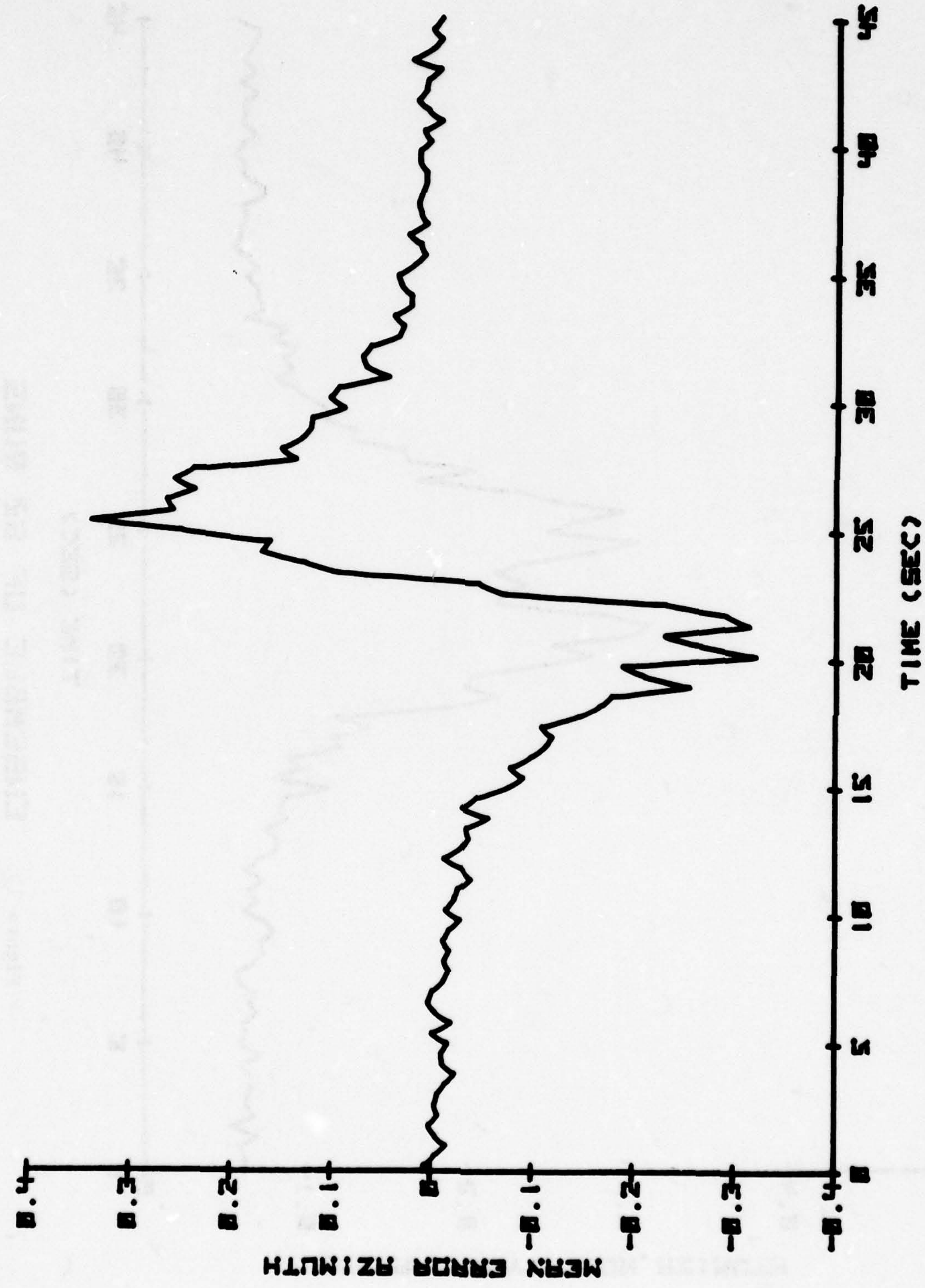


Figure 18. ENSEMBLE OF 60 RUNS

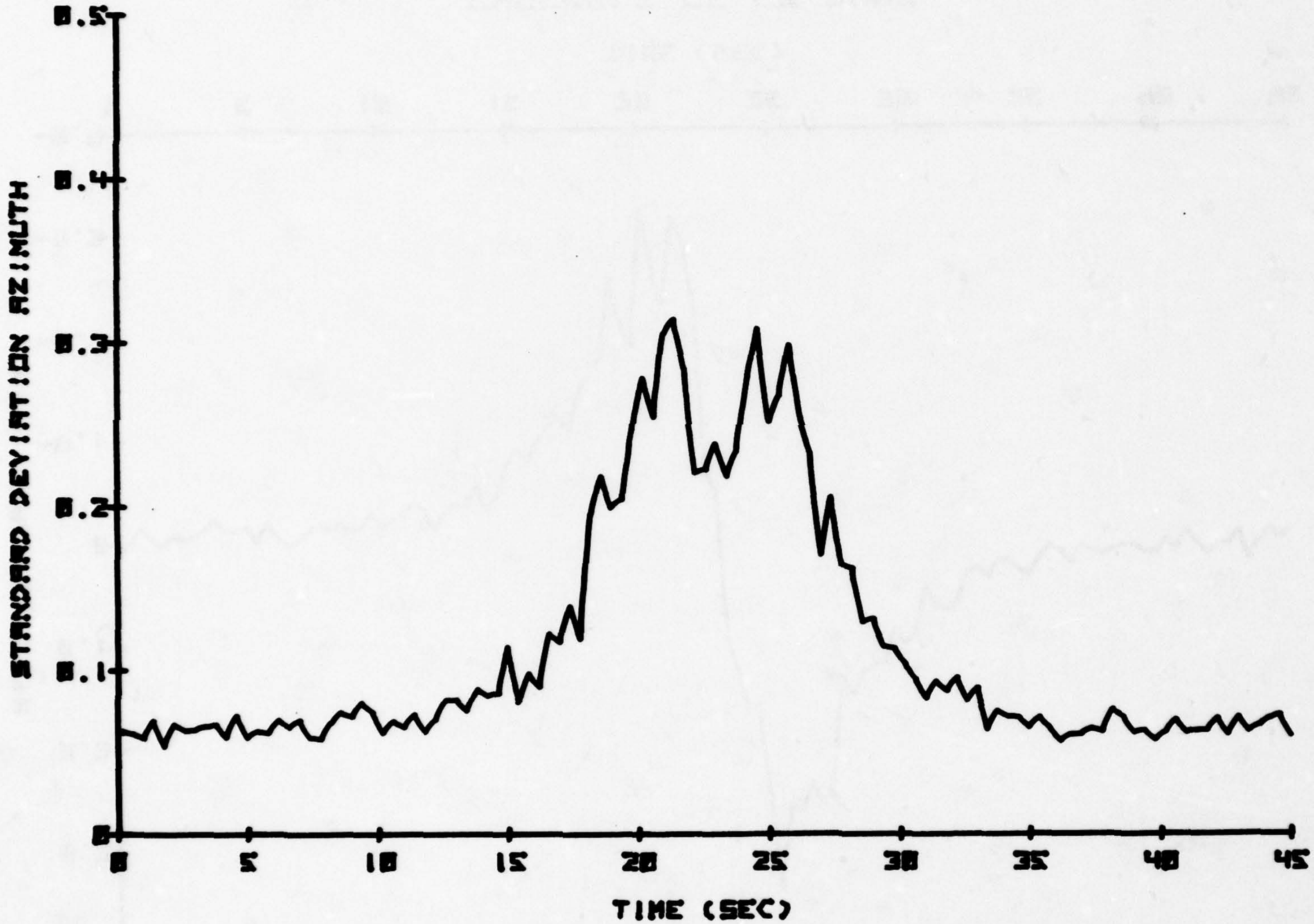


Figure 19. ENSEMBLE OF 60 RUNS

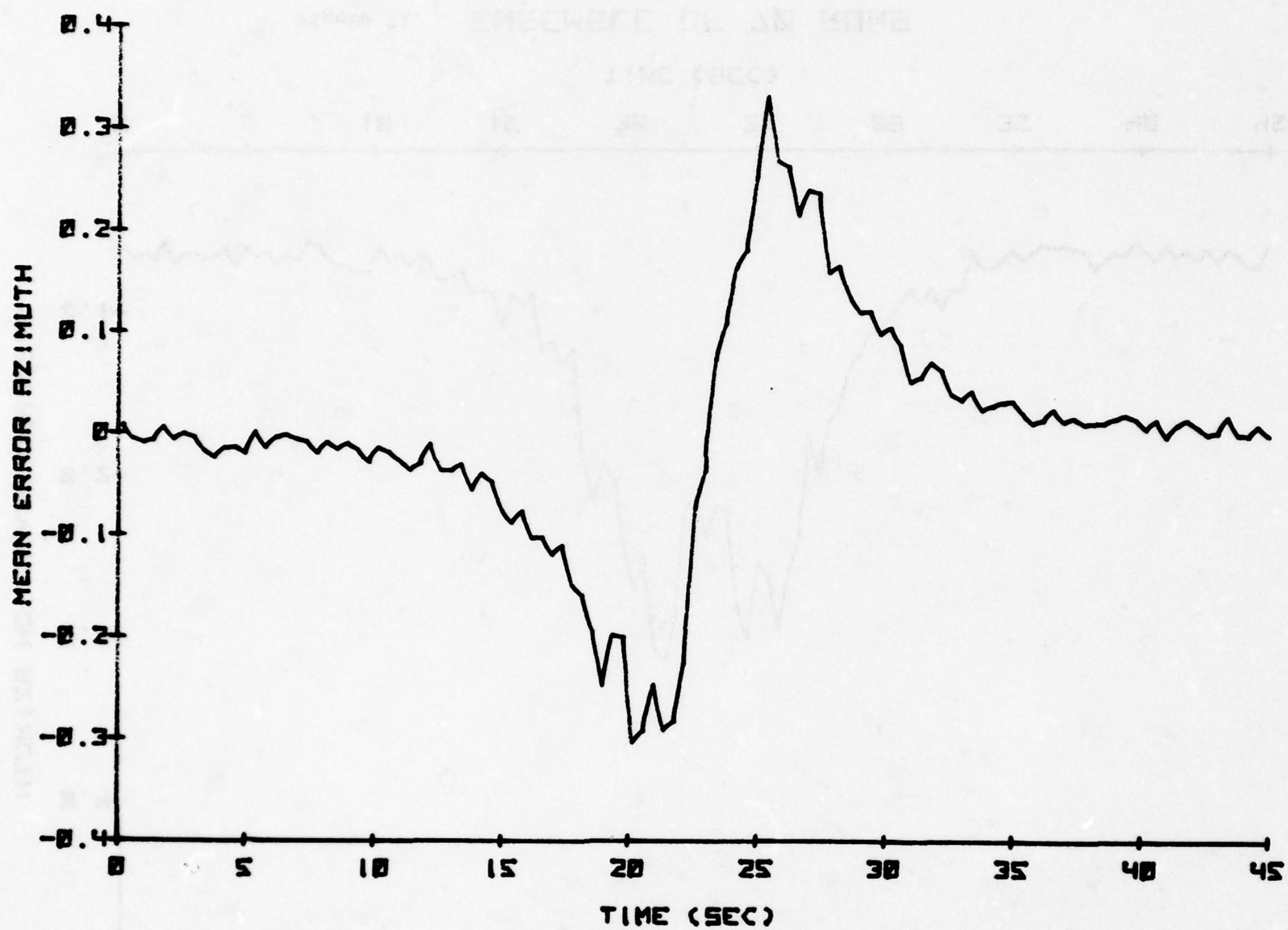


Figure 20. ENSEMBLE OF 70 RUNS

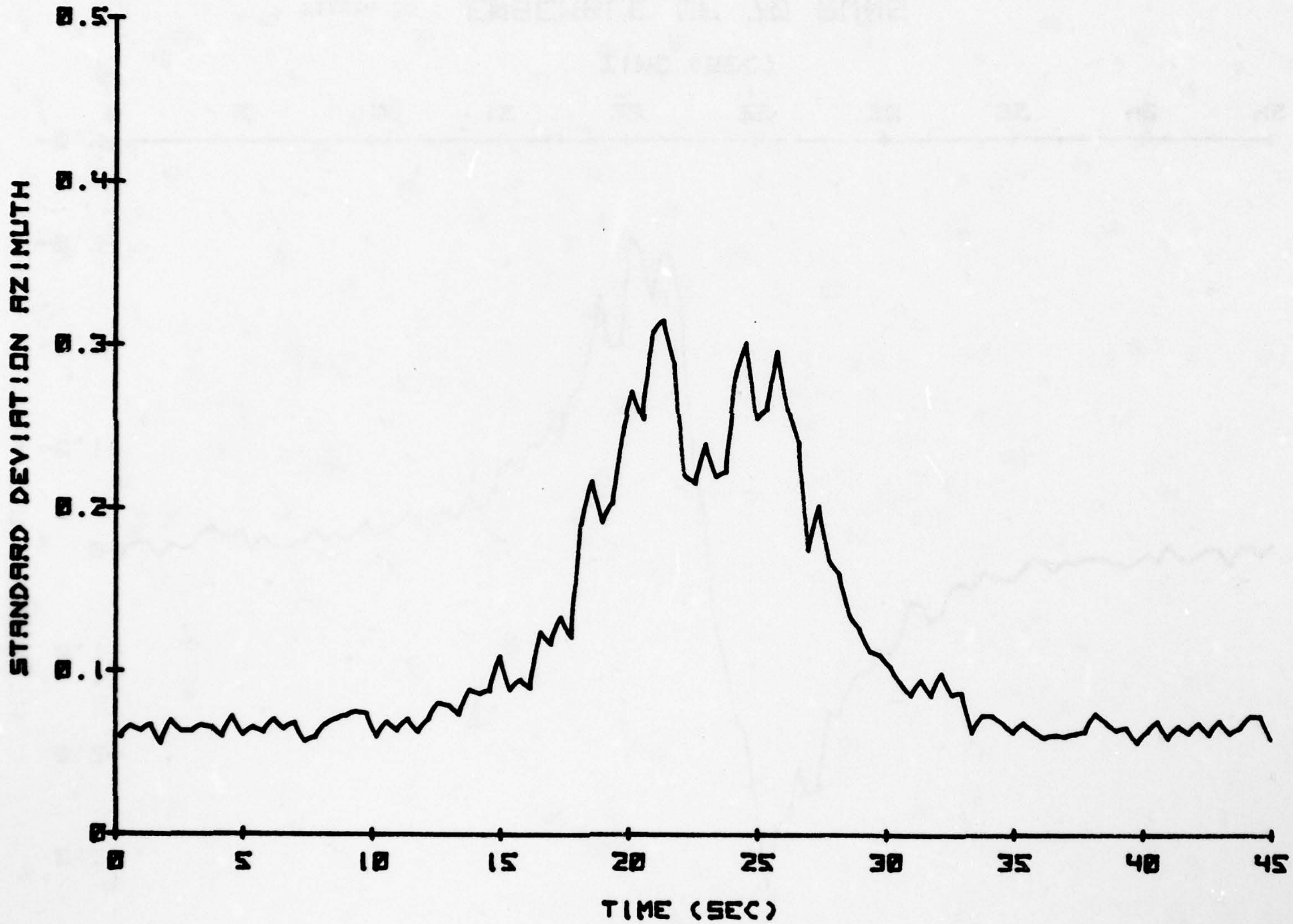


Figure 21. ENSEMBLE OF 70 RUNS

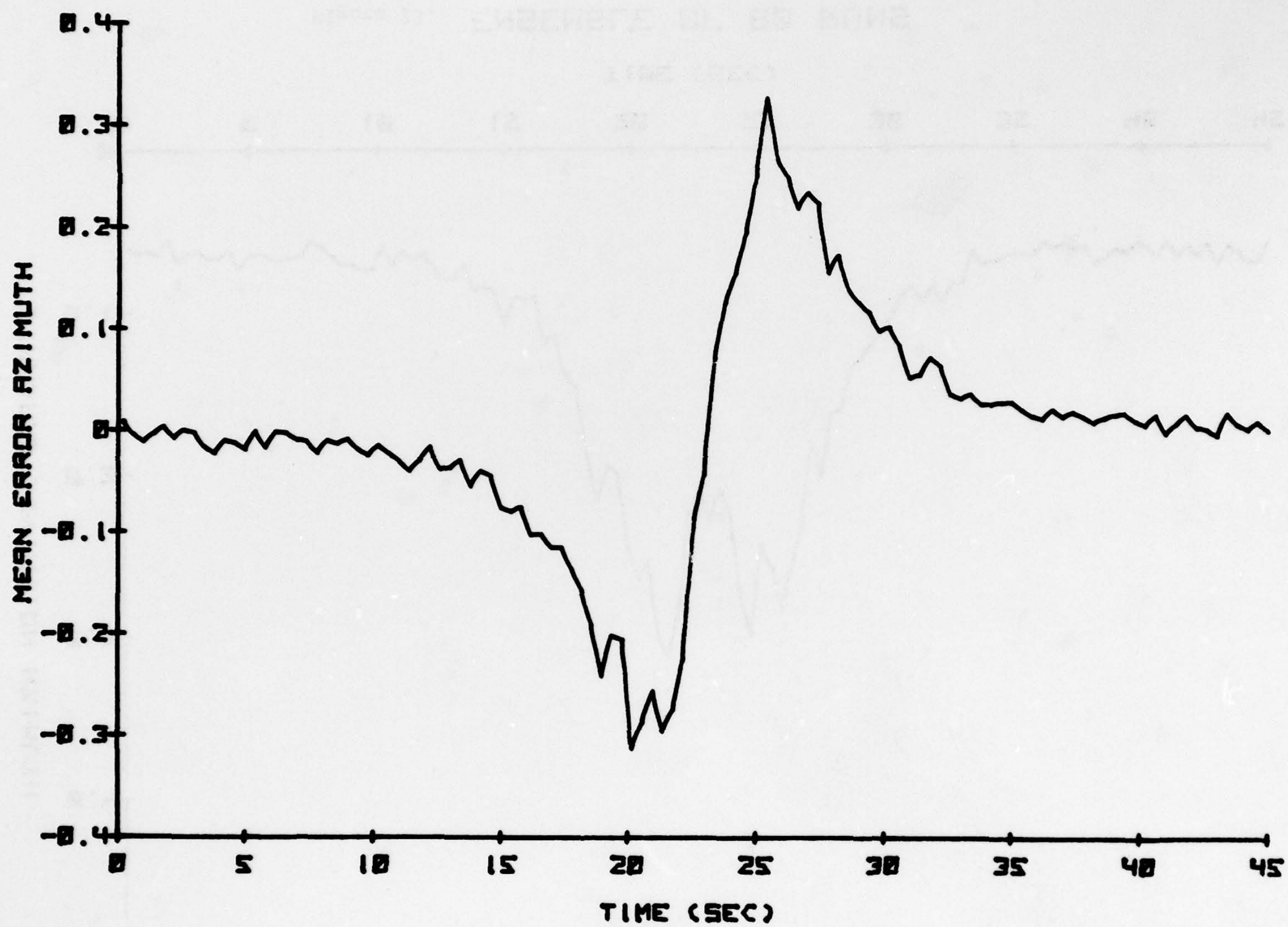


Figure 22. ENSEMBLE OF 80 RUNS

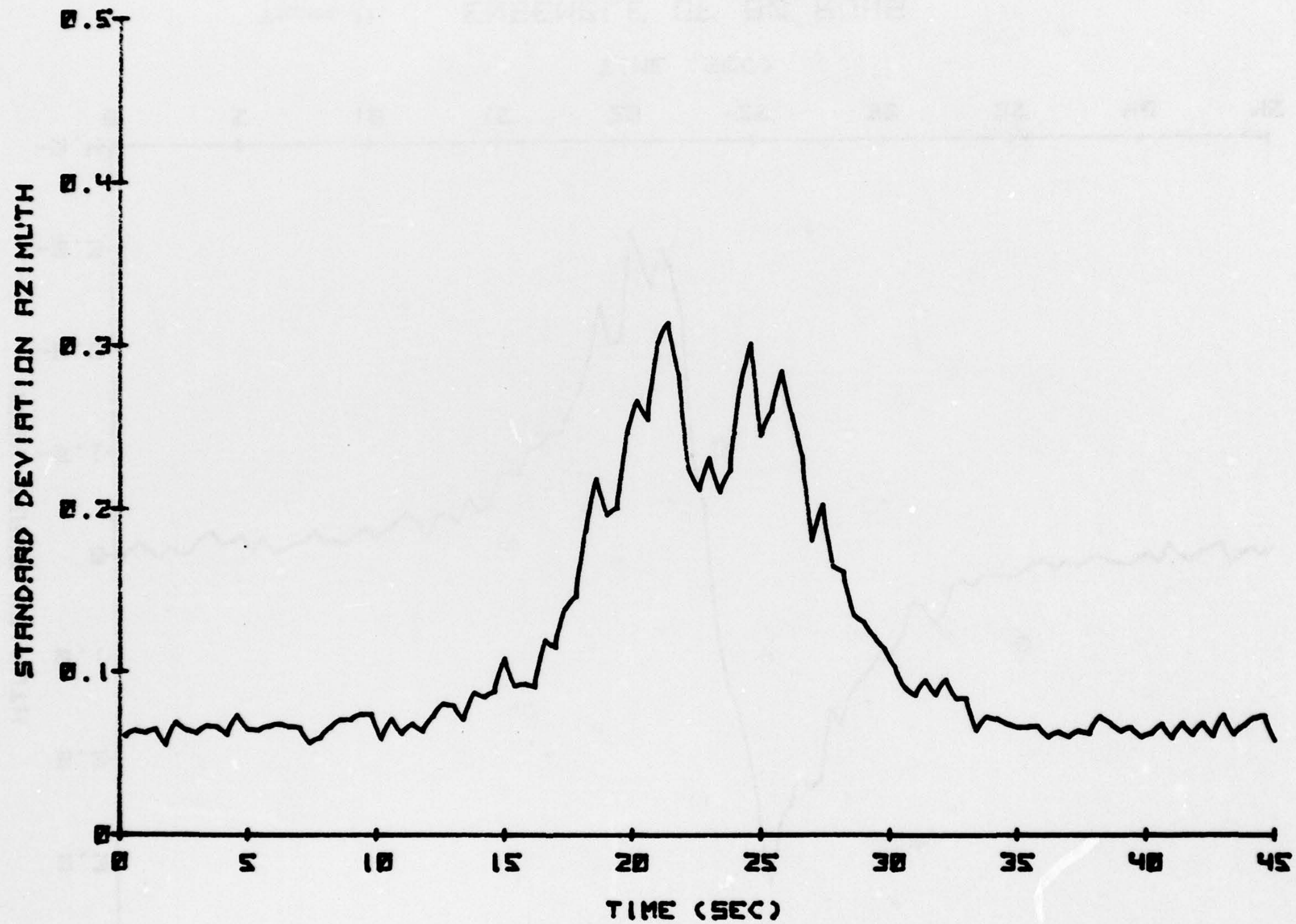


Figure 23. ENSEMBLE OF 80 RUNS

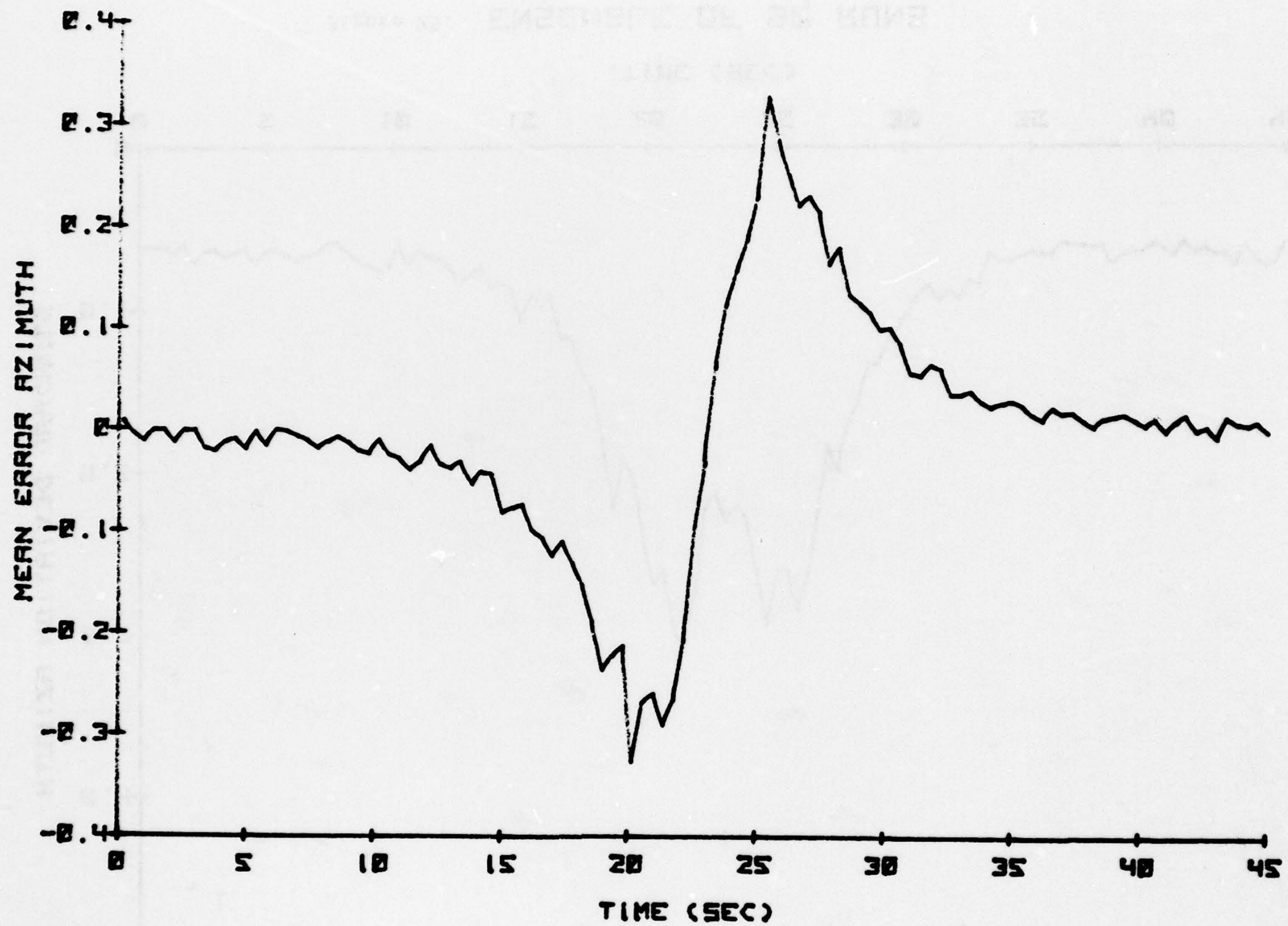


Figure 24. ENSEMBLE OF 90 RUNS

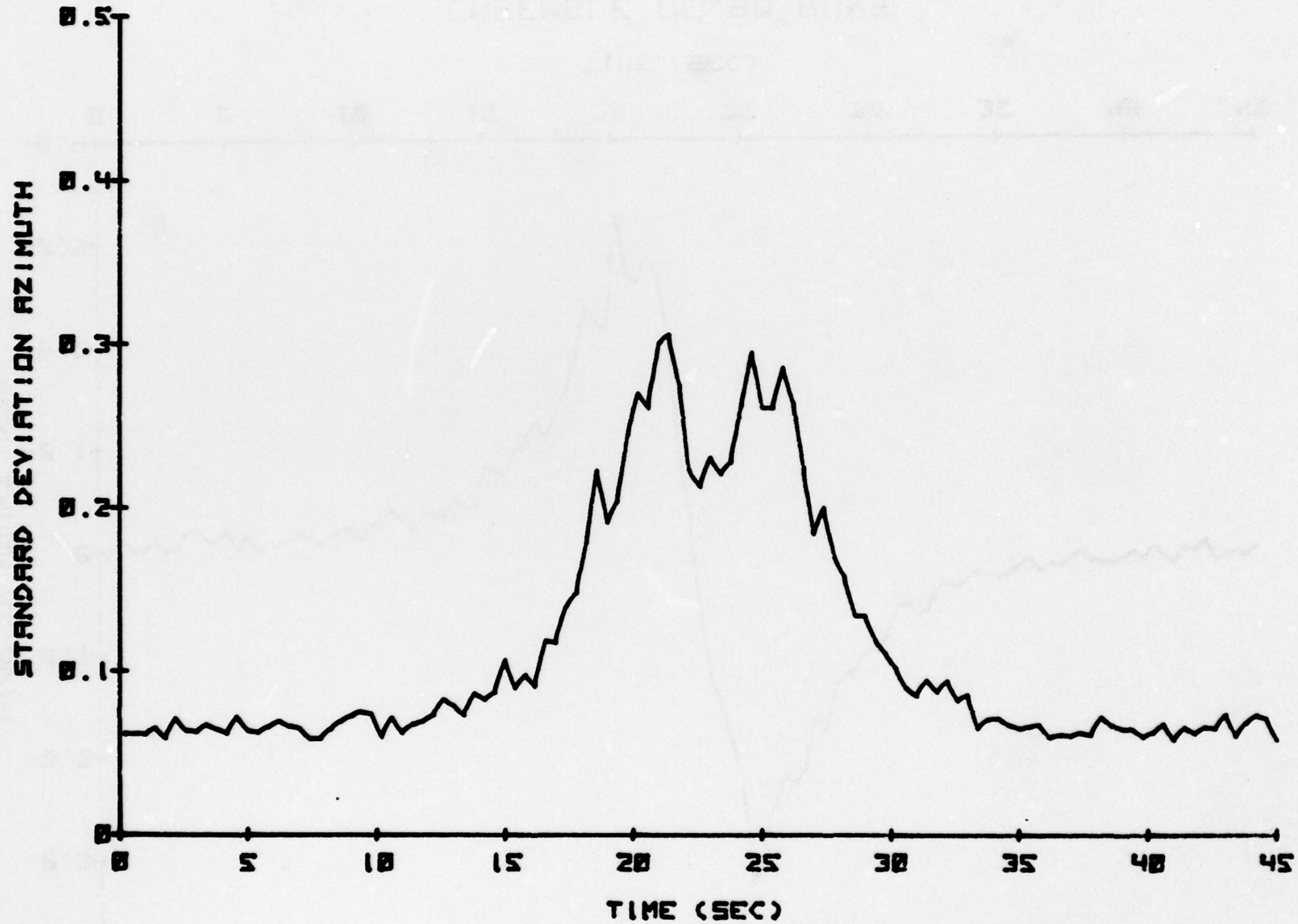


Figure 25. ENSEMBLE OF 90 RUNS

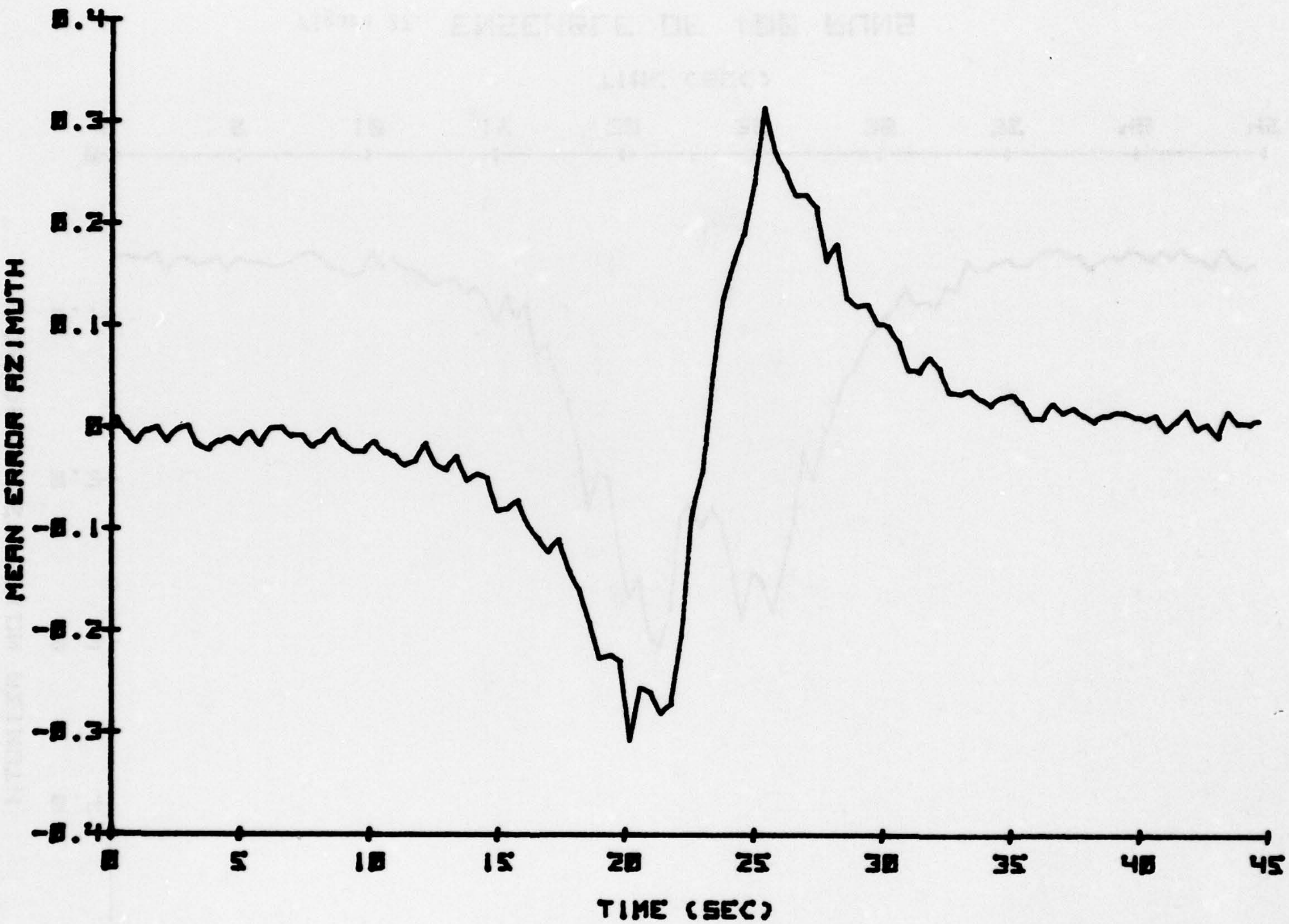


Figure 26. ENSEMBLE OF 100 RUNS

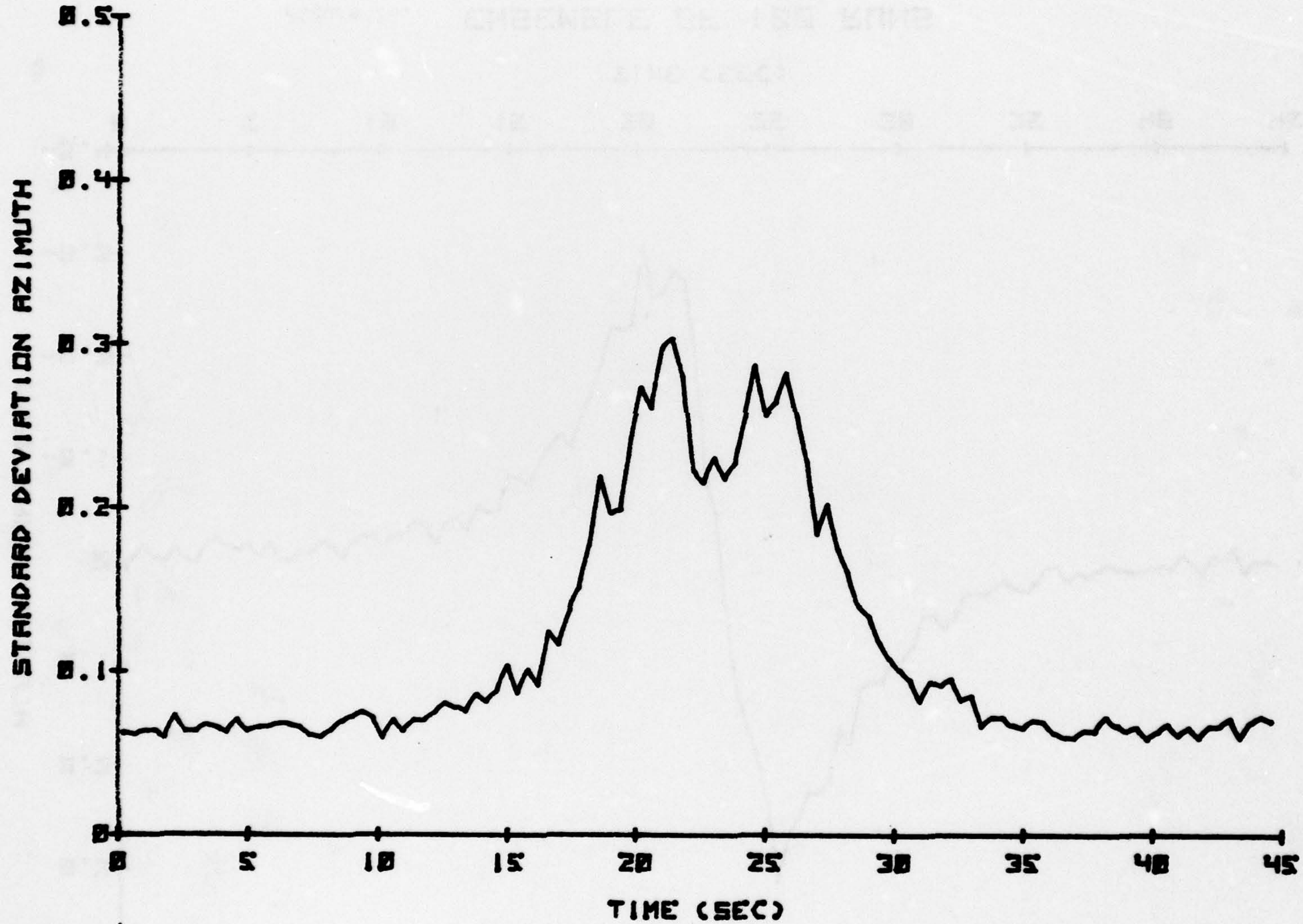


Figure 27. ENSEMBLE OF 100 RUNS

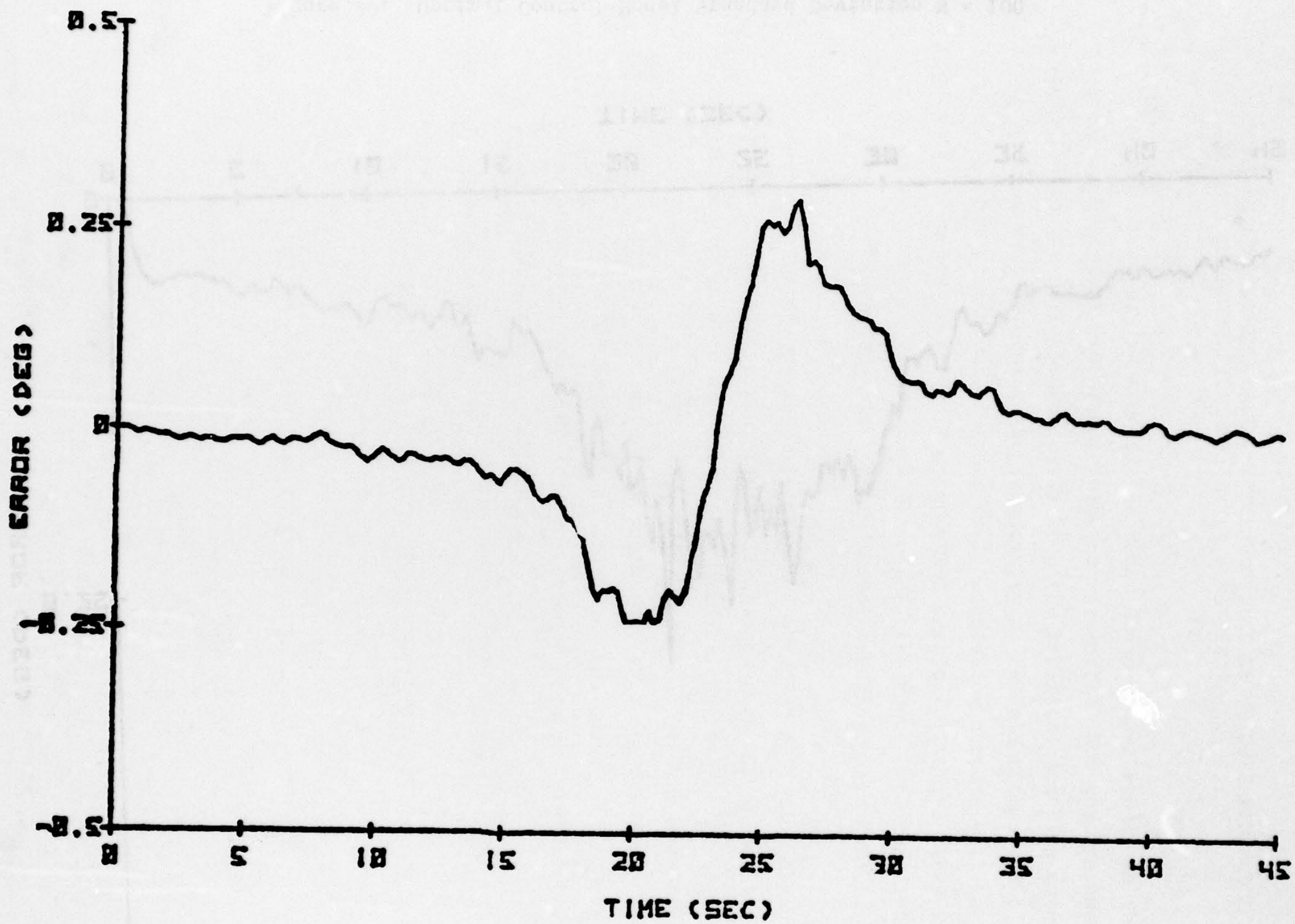


Figure 28. Optimal Control Model Ensemble Average N = 100

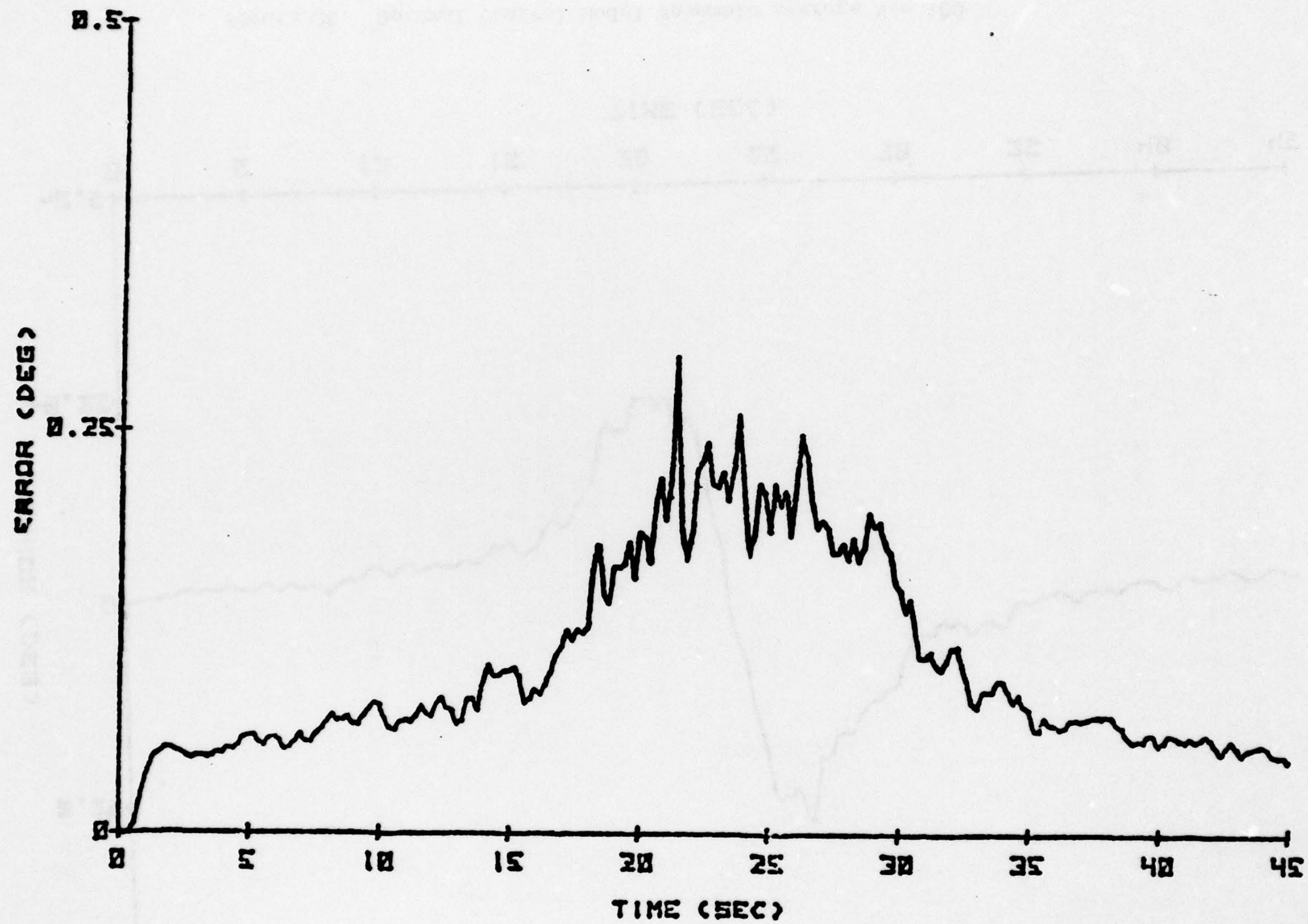


Figure 29. Optimal Control Model Standard Deviation N = 100

Section V

APPLICATION OF MONTE CARLO SIMULATION RESULTS TO OBSERVER MODEL PARAMETER IDENTIFICATION SENSITIVITY ANALYSIS

An important application of the Monte Carlo simulation is in the analysis of empirical data sample size requirements. Now that parameter identification is performed directly from the empirical tracking data, it is necessary to determine the sensitivity of the algorithms. The sample ensemble of individual Monte Carlo simulation outputs will be used as a reference to select the empirical data sample size. This is a valid approach since it has been shown that the Monte Carlo predictions are in very good agreement with the empirical tracking error data.

A description of the ensemble mean and standard deviation observer model parameter identification programs is given in detail in Refs. (1), (2). The sample ensemble of Monte Carlo predictions for n = 10 to n = 100 in increments of 10 runs were each used as input data to the mean parameter identification program. (Same flyby trajectory as mentioned previously in the report). The resultant parameter values and curve fitting cost functional (JMIN) are given in Table 2. Parameter values are determined iteratively by minimizing the following:

$$J = \int_0^{t_f} \left(\overline{e}_T - \overline{e}^1_T(t,\underline{\alpha}) \right)^2 dt$$

where,

$\overline{e}_T(t)$ = mean input tracking error data

$\overline{e}^1_T(t,\underline{\alpha})$ = predicted mean tracking error

And, $\underline{\alpha}$ is the parameter vector of interest. JMIN is the final value of the cost functional following completion of the iteration process.

JMIN as a function of sample size is plotted in Figure 30. From Figure 30 it can be seen that considerable improvements in fit occur as the sample size is increased to around 40 or 50 runs. Improvements become asymptotically smaller with even larger sample sizes. This result indicates that a minimum sample of about 40 or 50 runs is required for good parameter identification convergence properties.

TABLE 2. EFFECT OF SAMPLE SIZE ON OBSERVER MODEL PARAMETER IDENTIFICATION SENSITIVITY

NUMBER OF RUNS n	OBSERVER GAIN k	CONTROLLER GAIN x	JMIN
10	2.90	-2.90	1.77
20	2.87	-2.88	.902
30	2.73	-2.60	1.10
40	2.88	-2.89	.460
50	2.83	-2.69	.553
60	2.83	-2.80	.450
70	2.85	-2.79	.390
80	2.86	-2.80	.323
90	2.87	-2.81	.279
100	2.87	-2.90	.205

A similar study has been conducted using the standard deviation model parameter identification program. Figure 31 is a plot of JMIN versus sample size using the Monte Carlo simulation standard deviation results as input data. This graph also indicates that maximum improvement in the curve fitting cost functional is obtained as the number of ensembled runs is increased to 50 approximately. The above results were verified for both the mean and standard deviation identification programs using simulation data from another flyby trajectory.

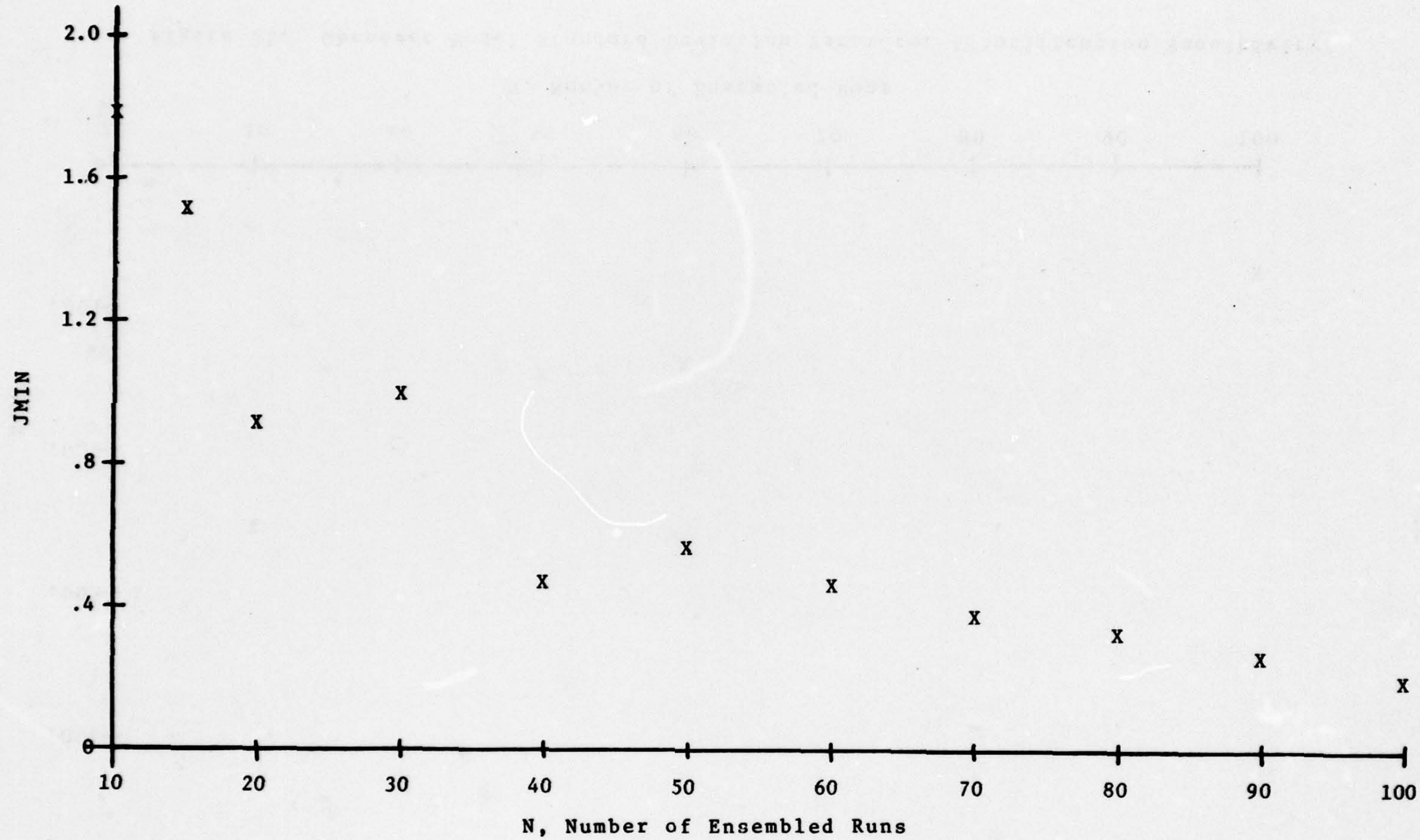


Figure 30. Observer Model Mean Parameter Identification Sensitivity

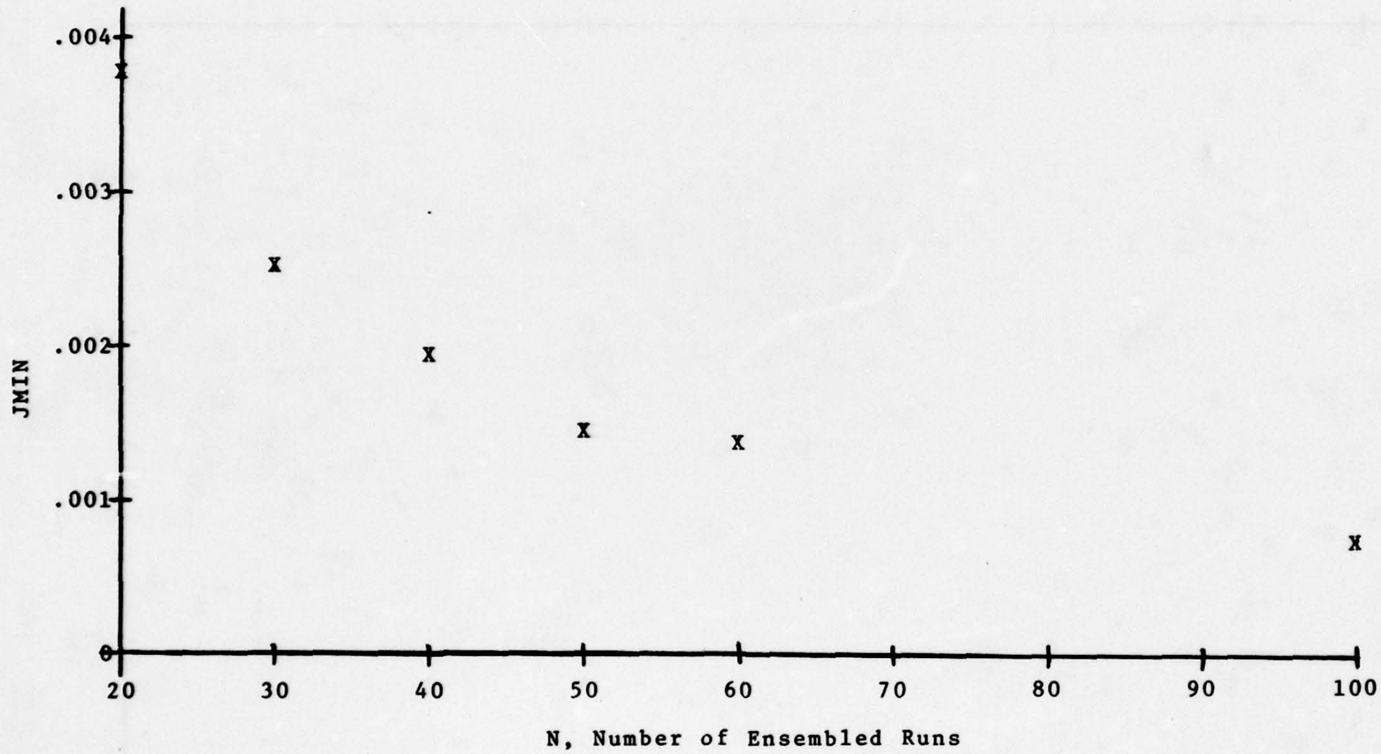


Figure 31. Observer Model Standard Deviation Parameter Identification Sensitivity

Section VI

CONCLUSION

Comparison of the sample ensemble of many Monte Carlo runs shows excellent agreement with the ensemble mean and standard deviation model outputs. The statistics converge especially well for n = 100 runs as the figures show. These results verify the validity of this simulation approach and computer program.

The Monte Carlo simulation of the observer model can be used with confidence in overall AAA system attrition analysis. Outputs from this human operator simulation can be readily integrated with other Monte Carlo anti-aircraft-artillery system components. The convergence analyses conducted herein indicate the number of runs necessary to obtain desired fidelity in the final attrition estimates.

Similation results have also been applied to study the sensitivity of the observer model parameter identification programs. Analyses indicate that a sample size of 40 to 50 runs gives good fidelity in the curve fitting programs. This information will be used in the design of tracking experiments that generate data for modeling purposes.

REFERENCES

(1) R. S. Kou and B. C. Glass, Development of Observer Model for AAA Tracker Response, AMRL-TR-79-77 (in press), Aerospace Medical Research Laboratory, Wright-Patterson Air Force Base, Ohio, August 1979

(2) R. S. Kou, B. C. Glass, C. N. Day, and M. M. Vikmanis, "AAA Gunner Model based on Observer Theory," presented at the 14th Annual Conference on Manual Control at Los Angeles, California, April 25 through 27, 1978.

(3) D. L. Kleinman, Monte-Carlo Simulation of Human Operator Response, University of Connecticut Technical Report, TR 77-1, February, 1977.

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